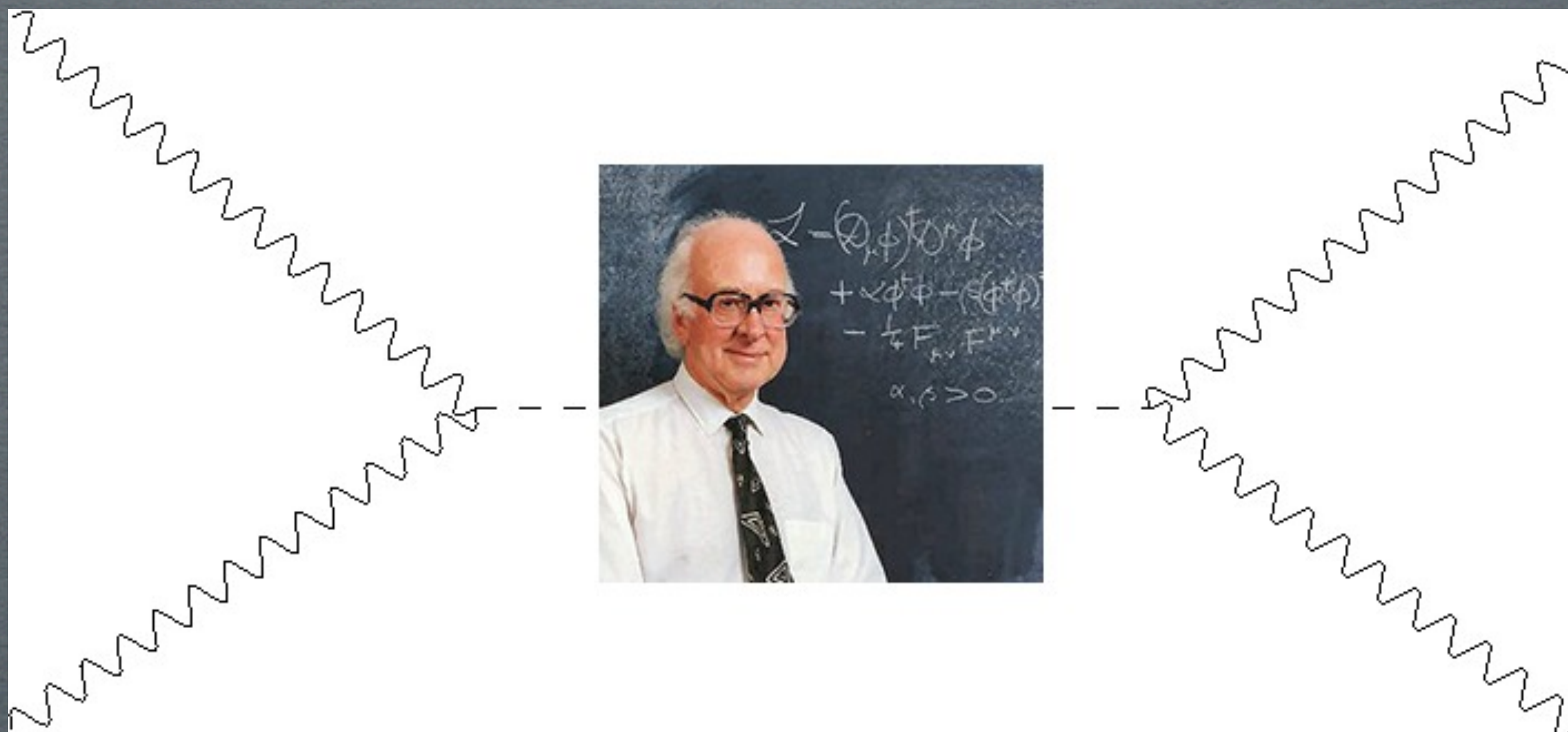
The Matrix Element Method For Higgs Discovery



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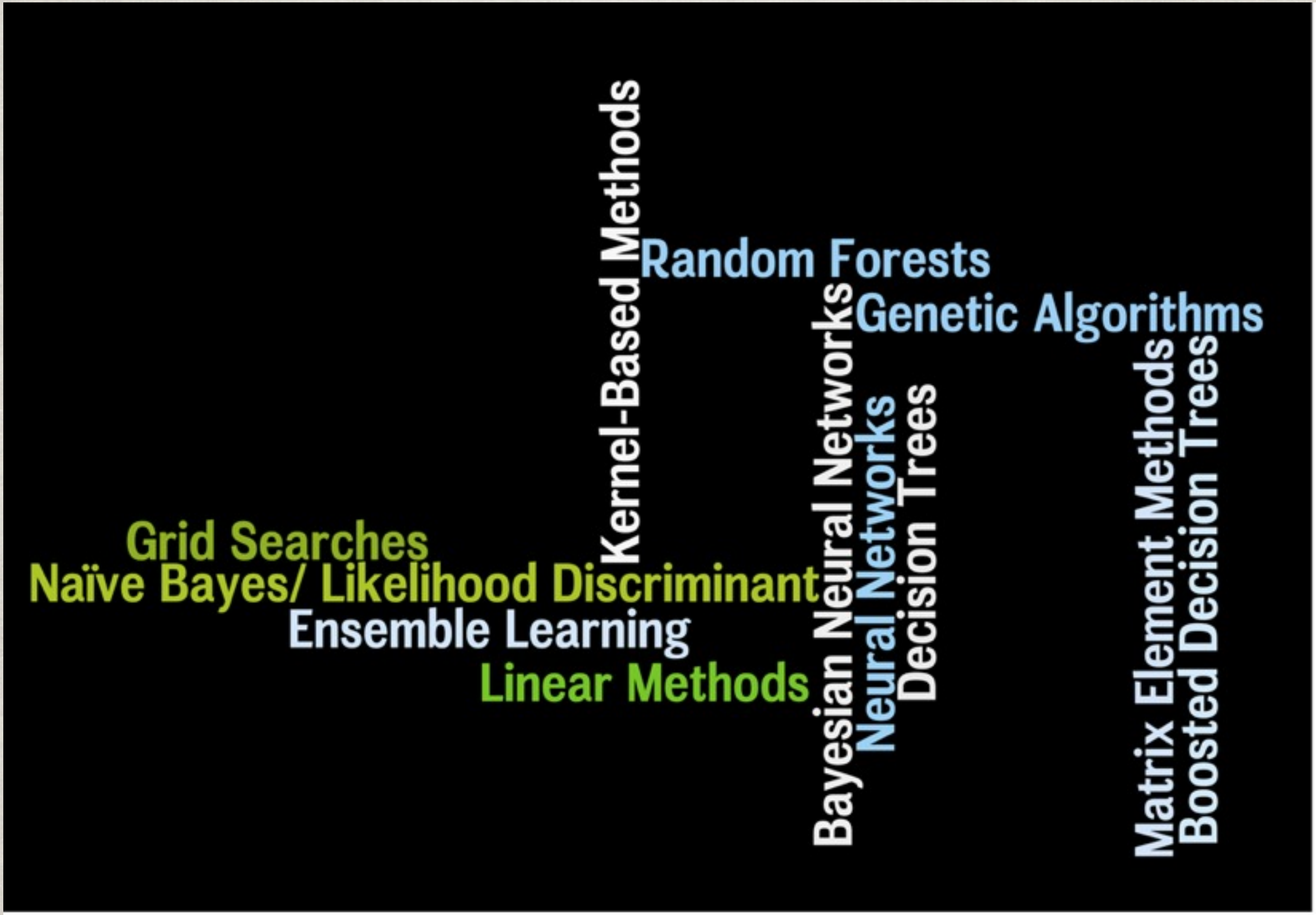
Outline

- The Matrix Element Method (MEM)
- Previous use of MEM for Higgs in ZZ ➔ 4l
- Improvement of discovery significance for Higgs in ZZ ➔ 4l using the MEM at the 7 TeV LHC (arXiv:1108.2274 [hep-ph]).
- Future Directions

Multivariate analyses



wordle.com

c.f. Bhat, FERMILAB-PUB-10-054-E

Matrix Element Method

- The Matrix Element Method (MEM) = use of the matrix element/ differential cross section as a likelihood function

- In our analyses, we will use the expression for likelihood:

$$\mathcal{L}(\mu;\boldsymbol{\theta}) = \frac{e^{-\mu}\mu^N}{N!}\prod_{i=1}^{N} P(\boldsymbol{\theta}; x_i)$$

Matrix Element Method

$$\mathcal{L}(\mu;\boldsymbol{\theta}) = \frac{e^{-\mu}\mu^N}{N!}\prod_{i=1}^{N} P(\boldsymbol{\theta};x_i)$$

- μ is the expected number of events
- N is the observed number of events
- P is the normalized differential cross section as a function of the underlying model and model parameters (**θ**) and the properties of the event (momenta of observed particles, x_i)

Matrix Element Method

- In general, P is not simply the differential cross section for the observed momenta
  - Some 4-momenta may be poorly measured (esp. jets), should integrate over transfer function
  - Some processes may involve final state particles which are not observed (neutrinos, neutralinos, etc.). Need to integrate over undetermined momenta.

Matrix Element Method

- Tools exist to make integration over undetermined momenta easier (MadWeight)
- though additional integration unavoidably makes the MEM computation intensive- can be deterrent to experimentalists.
- Can avoid transfer functions (more or less) and integrations over unobserved particles by considering final states with only charged leptons and photons.

ZZ Final States

- This makes the ZZ final state, where both Zs decay leptonically and the final state can therefore be completely reconstructed, especially attractive for study.

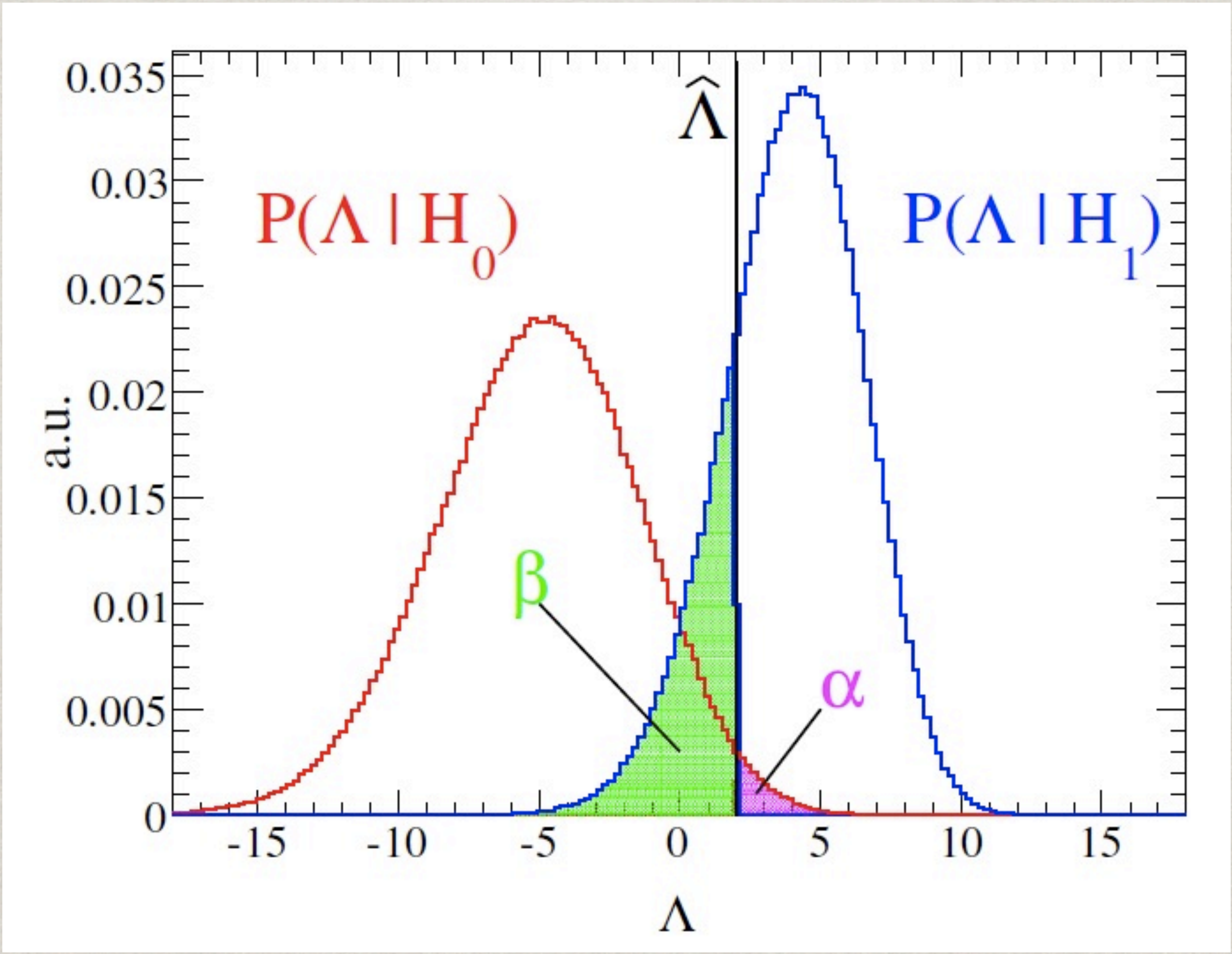
Higgs look-alikes at the LHC

- De Rujula, Lykken, Pierini, Rogan, Spiropulu, Phys. Rev. D82 (2010) 13003.
- Studied how effectively resonances decaying to ZZ with different spins, couplings, and CP properties can be distinguished at the 10 TeV LHC.
- Used $_{s}$Plots method, in which background events are effectively removed from distributions by reweighting each event.

Higgs look-alikes at the LHC

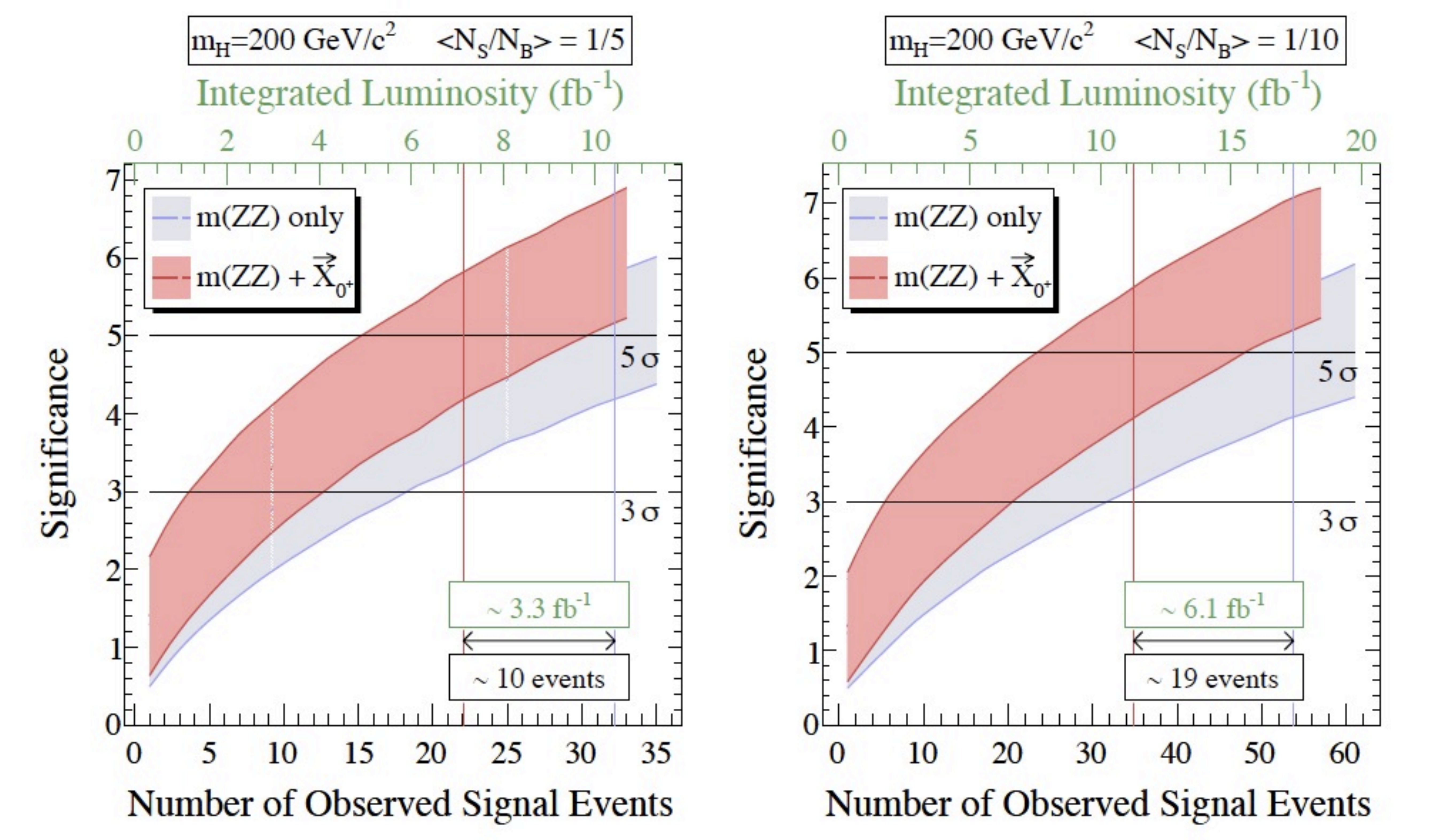
- Considered full angular correlations (via the MEM) as well as invariant mass, etc.
- Generally found different resonances could be distinguished with a relatively low number of events.
- Also looked at Higgs discovery significance at specific masses (still at 10 TeV) using invariant mass information.

Higgs look-alikes at the LHC

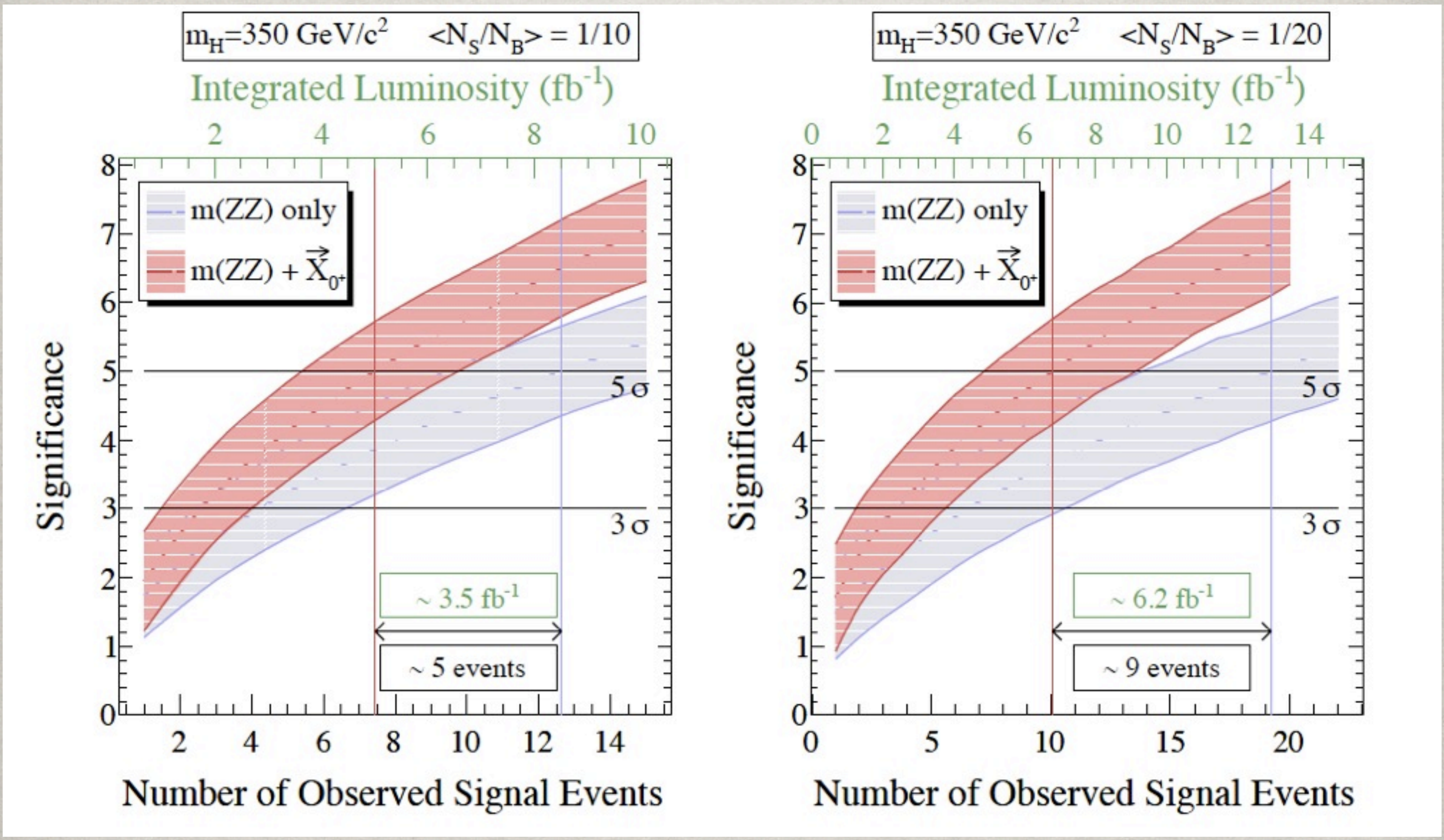


$H_0 = 0^+$ vs. $H_1 = 0^-$, $m_h = 200$ GeV, $N_s = 23$

Higgs look-alikes at the LHC



Higgs look-alikes at the LHC



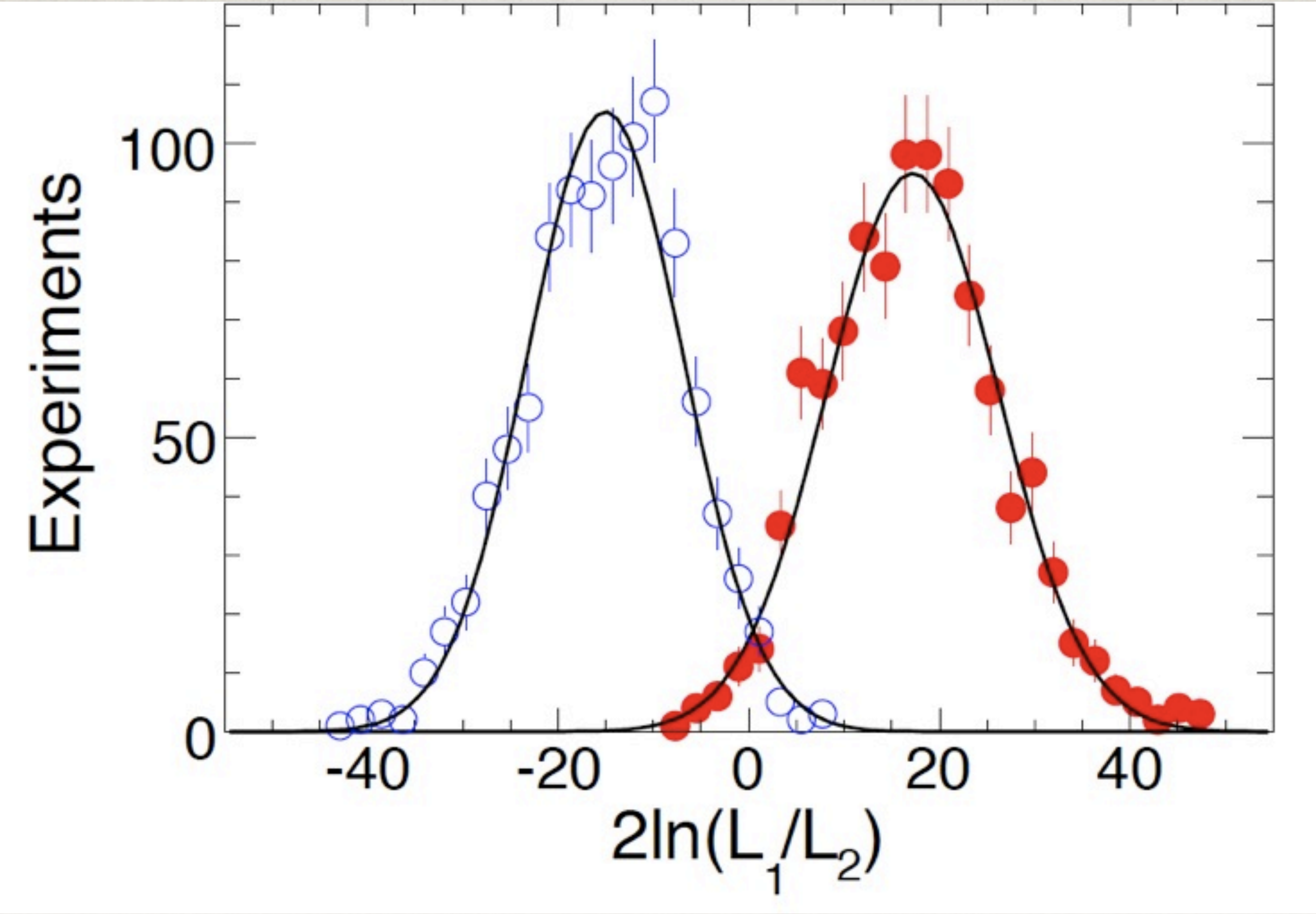
Spin determination of single-produced resonances at hadron colliders

- Gao, Gritsan, Guo, Melnikov, Schulze, Tran, Phys. Rev. D81 (2010) 075022.
- Studied how effectively resonances decaying to ZZ with different spins, couplings, and CP properties can be distinguished at the 14 TeV LHC.

Spin determination of single-produced resonances at hadron colliders

- Used the matrix element method.
- Backgrounds from MadEvent.
- Generally found that different resonances could be clearly distinguished in this channel.

Spin determination of single-produced resonances at hadron colliders



$L_1 = 0^+$ vs. $L_1 = 0^-$, $m_h = 250$ GeV, $N_s = 30$

ZZ Discovery

- We (JG, Kunal Kumar, Ian Low, and Roberto Vega-Morales) wanted to understand how much the MEM could aid in Higgs discovery in ZZ final states at the 7 TeV LHC.

- arXiv:1108.2274 [hep-ph]

Our MEM/ Likelihood

- Use the MEM to distinguish signal and background.
- Specifically we generalize

$$\mathcal{L}(\mu;\boldsymbol{\theta}) = \frac{e^{-\mu}\mu^N}{N!}\prod_{i=1}^{N} P(\boldsymbol{\theta}; x_i)$$

Our MEM/ Likelihood

- to

$$\mathcal{L}_{s+b}(\mu, f, m_h) = \frac{e^{-\mu}\mu^N}{N!}\prod_{i=1}^{N}[f P_s(m_h; x_i) + (1-f)P_b(x_i)]$$

where

$$0 < f = \frac{\mu_s}{\mu_s + \mu_b} < 1$$

$P_{s(b)}$ = Normalized signal (background) differential cross section

$\mu_{s(b)}$ = expected signal (background) number of events

Our MEM/ Likelihood

- When we maximize the above expression with respect to μ, we obtain μ = N (the observed number of events).

- So we are left with

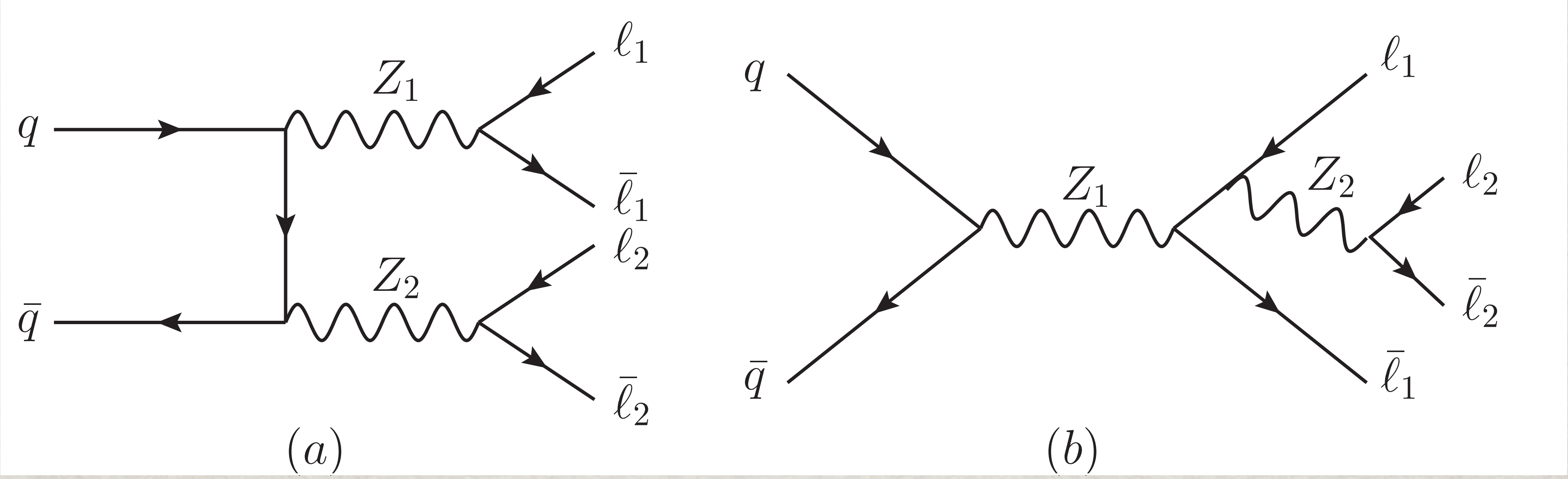
$$\prod_{i=1}^{N}[f P_s(m_h; x_i) + (1 - f)P_b(x_i)]$$

- To proceed further, we need expressions for P_s and P_b

Differential Cross Sections

- We obtain analytic expressions for $P_{s(b)}$
- Allows greater understanding of what is providing signal vs. background discrimination
- Built from helicity amplitudes

Backgrounds considered



- We consider only diagrams as in (a) (and its u-channel counterpart), at LO
- There are LO diagrams, like that shown in (b), which we also do not consider

Background Helicity amplitudes

- We express amplitudes for the production of Z bosons with definite helicity from initial (massless) fermions with definite helicity as

$$\mathcal{M}^{ZZ}_{\sigma\bar{\sigma};\lambda_1\lambda_2} = 4\sqrt{2}\left(g^{Zq\bar{q}}_{\Delta\sigma}\right)^2 \epsilon\,\delta_{|\Delta\sigma|,\pm 1}\frac{\mathcal{A}^{\Delta\sigma}_{\lambda_1\lambda_2}(\Theta)\; d^{J_0}_{\Delta\sigma,\Delta\lambda}(\Theta)}{4\beta_1\beta_2\sin^2\Theta + (1-\beta_1\beta_2)^2 - x^2(1+\beta_1\beta_2)^2}$$

where

$$\Delta\sigma = \sigma - \bar{\sigma},\ \epsilon = \Delta\sigma(-1)^{\lambda_2},\ \Delta\lambda = \lambda_1 - \lambda_2, \text{ and } J_0 = \max(|\Delta\sigma|, |\Delta\lambda|)$$

following the convention in Hagiwara, Hikasa, Peccei, and Zeppenfeld (Nucl.Phys. B282 (1987) 253)

Background Helicity amplitudes

$$\Delta\lambda = \pm 2: \quad \mathcal{A}^{\Delta\sigma}_{\pm\mp} = -\sqrt{2}(1+\beta_1\beta_2)\ ,$$

$$\Delta\lambda = \pm 1: \quad \mathcal{A}^{\Delta\sigma}_{\pm 0} = \frac{1}{\gamma_2(1+x)}\Big[(\Delta\sigma\Delta\lambda)\Big(1+\frac{\beta_1^2+\beta_2^2}{2}\Big) - 2\cos\Theta$$
$$-(\Delta\sigma\Delta\lambda)(\beta_2^2-\beta_1^2)x - 2x\cos\Theta - (\Delta\sigma\Delta\lambda)\Big(1-\frac{\beta_1^2+\beta_2^2}{2}\Big)x^2\Big]$$

$$: \quad \mathcal{A}^{\Delta\sigma}_{0\pm} = \frac{1}{\gamma_1(1-x)}\Big[(\Delta\sigma\Delta\lambda)\Big(1+\frac{\beta_1^2+\beta_2^2}{2}\Big) - 2\cos\Theta$$
$$-(\Delta\sigma\Delta\lambda)(\beta_2^2-\beta_1^2)x + 2x\cos\Theta - (\Delta\sigma\Delta\lambda)\Big(1-\frac{\beta_1^2+\beta_2^2}{2}\Big)x^2\Big]$$

$$\Delta\lambda = 0: \quad \mathcal{A}^{\Delta\sigma}_{\pm\pm} = -(1-\beta_1\beta_2)\cos\Theta - \lambda_1\Delta\sigma(1+\beta_1\beta_2)x\ ,$$

$$\Delta\lambda = 0: \quad \mathcal{A}^{\Delta\sigma}_{00} = 2\gamma_1\gamma_2\cos\Theta\Big[((1-x)\beta_1 + (1+x)\beta_2)\sqrt{\frac{\beta_1\beta_2}{1-x^2}} - (1+\beta_1^2\beta_2^2)\Big]$$

- Note that all helicity amplitudes depend only on a production angle Θ and that different helicity amplitudes have different energy dependences

Signal Helicity amplitudes

- For signal we consider gluon fusion production of the Higgs and then its decay to Z bosons with definite helicity (also LO)

$$\mathcal{M}^{ZZ}_{h;\pm1\pm1} = \frac{\alpha_s m_Z^2 \hat{s}}{3\pi v^2((\hat{s}-m_h^2)^2 + m_h^2\Gamma_h^2)^{1/2}} \,,$$

$$\mathcal{M}^{ZZ}_{h;00} = \gamma_1\gamma_2(1+\beta_1\beta_2)\frac{\alpha_s m_Z^2 \hat{s}}{3\pi v^2((\hat{s}-m_h^2)^2 + m_h^2\Gamma_h^2)^{1/2}}$$

- Helicities of Zs are equal for all non-vanishing amplitudes.
  - Due to the Higgs being spin-0.

Z Decay Amplitudes

- The amplitude for a Z with a particular helicity to decay to leptons is

$$\mathcal{M}^{(i)}_{\lambda_i;\sigma_i\bar{\sigma}_i} = \Delta\sigma_i \, (-1)^{\lambda_i}\sqrt{2} \; g^{Z\ell\bar{\ell}}_{\Delta\sigma} \; d(\Delta\sigma_i, \lambda_i, \theta_i) \; m_i \; e^{i\lambda_i\phi_i}$$

where

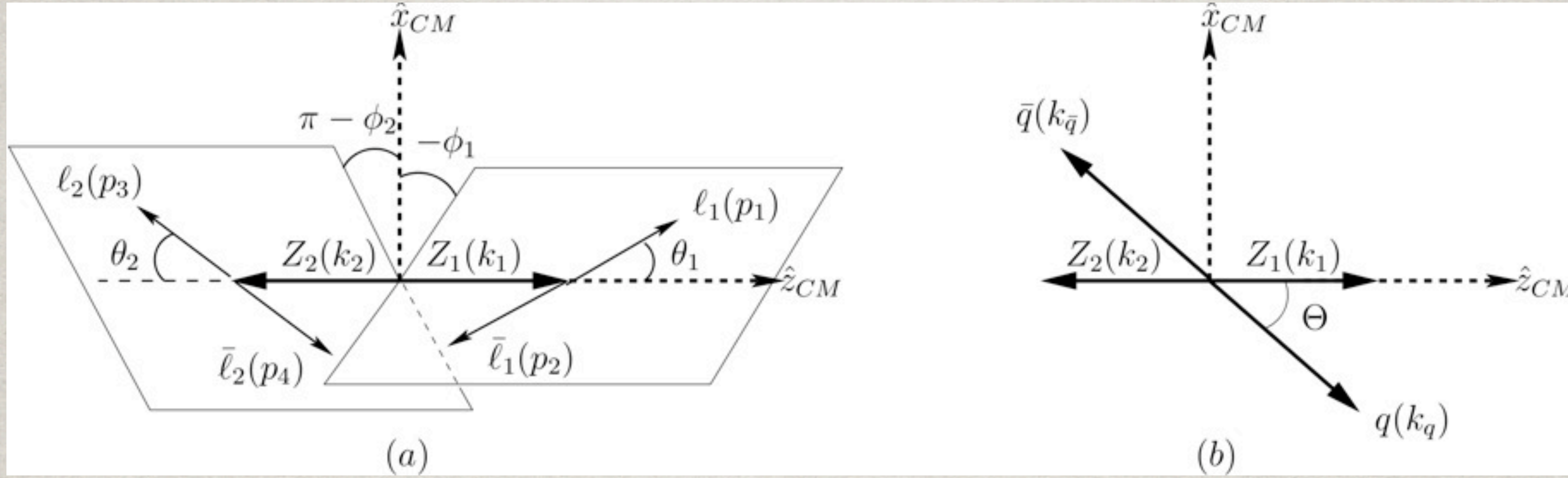
$$d(\Delta\sigma_i, \lambda_i, \theta_i) = d^{\max(\ |\Delta\sigma_i|,\ |\lambda_i|\ )}_{\Delta\sigma_i,\ \lambda_i}(\theta_i)$$

- Note the dependence on θ_i and ϕ_i.
Thus 4 of the 5 angles describing the event are essentially measuring the helicities of the Zs.

Differential Cross Sections

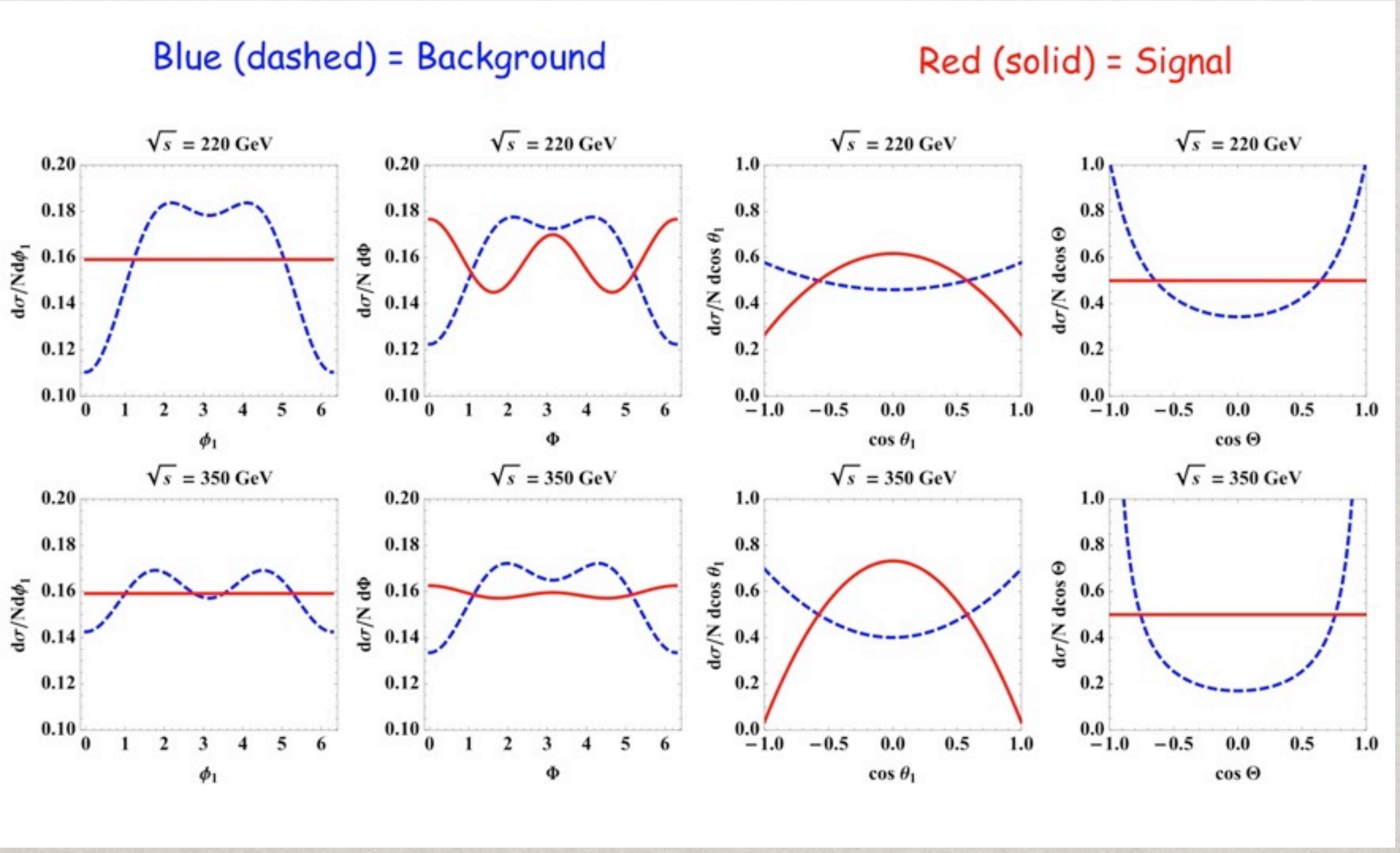
- With these production and decay helicity amplitudes it is straightforward to calculate the matrix element for signal or background production of a given event.
- Need to sum over directions of incoming quark (as opposed to antiquark) for background.
- Also include PDFs, phase space factor, etc.

Angular convention

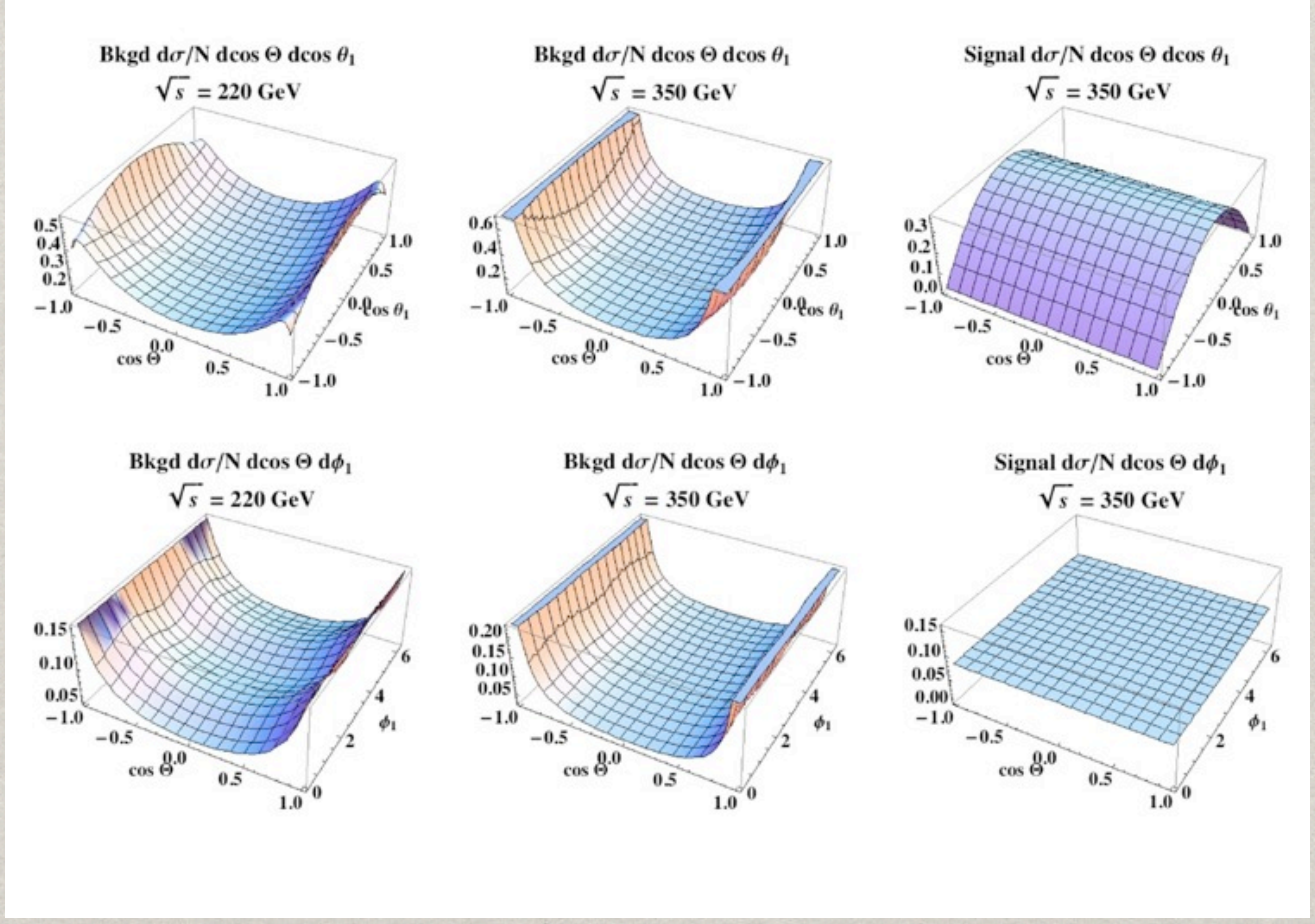


- The angles in the amplitudes above are defined in a convention where Θ is the angle between Z_1 and the z (initial quark in background case) axis in the CM frame.
- $\theta_{1(2)}$ and $\phi_{1(2)}$ are the standard θ and ϕ angles in the frame obtained by
  - Boosting along the z-axis from the lab frame to the CM frame
  - Rotating about the y-axis so that $Z_{1(2)}$ is in the z direction
- Our paper also contains Lorentz invariant expressions for these angles

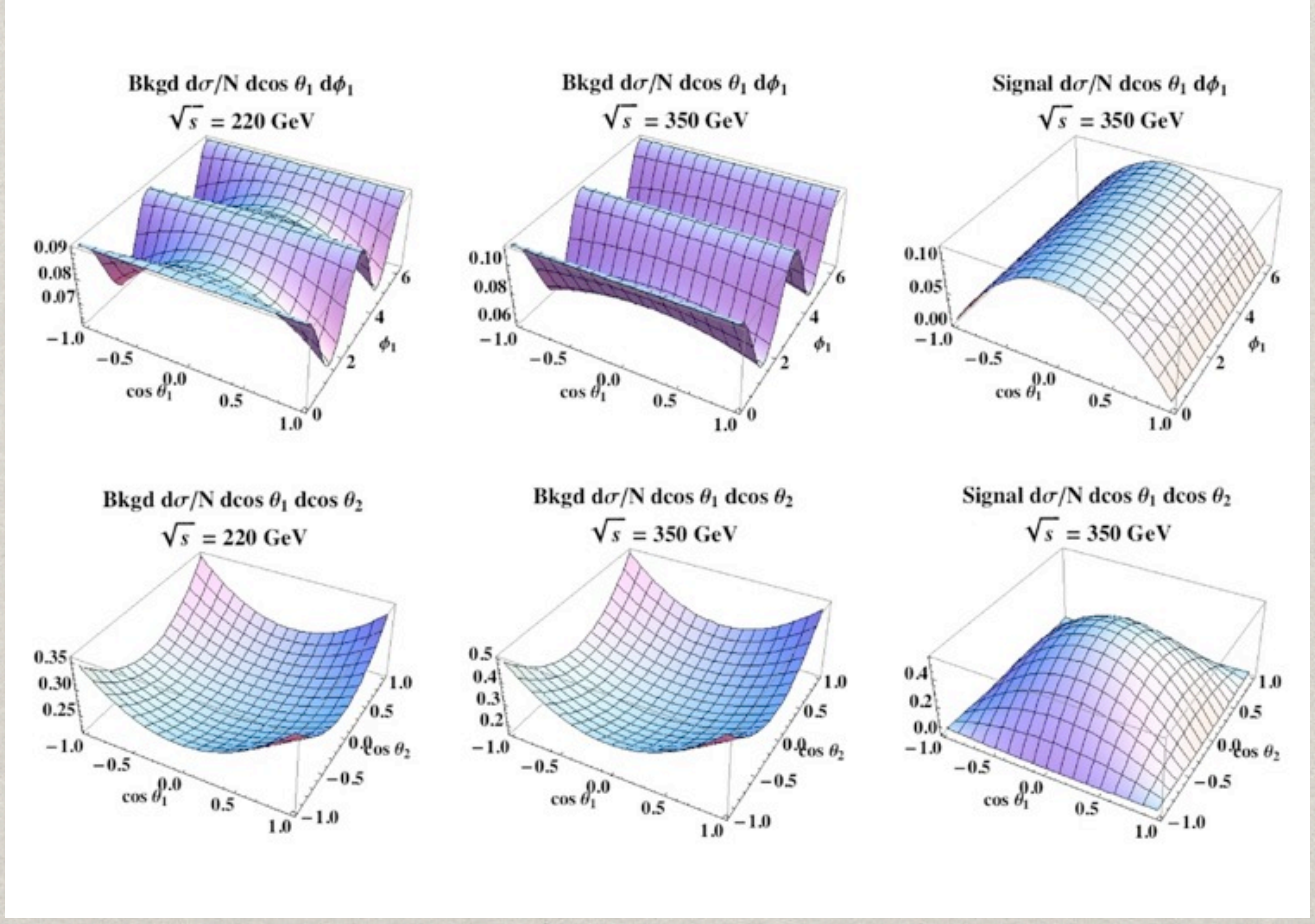
Angular Distributions



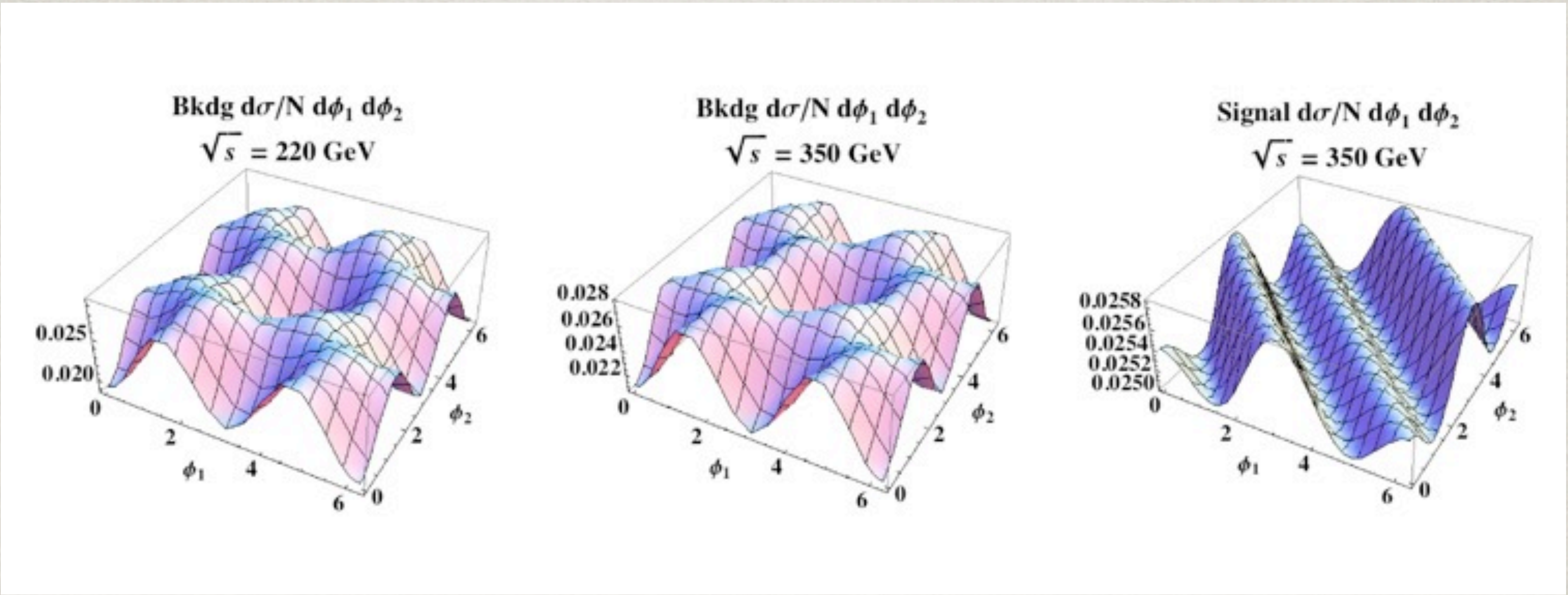
Doubly-Differential Angular Distributions



Doubly-Differential Angular Distributions



Doubly-Differential Angular Distributions



- Much more information in 2-D distributions than 1-D
- Presumably still more in 5 angles + M_1, M_2, x_1, x_2 = 9-D distributions.
- (x_1, x_2 can be replaced with event invariant mass and pseudo-rapidity)

Smearing/ Acceptance

- However, things are made somewhat more complicated by detector smearing, the effect of detector geometry, and the effect of cuts.

- We smear electron energies according to

$$\left(\frac{\sigma_{E,e}}{E}\right)^2 = \left(\frac{0.036}{\sqrt{E}}\right)^2 + 0.0026^2$$

and muon pT according to

$$\sigma_{p_T,\mu} = 0.015\ p_T - 5.710^{-6}\ p_T^2 + 2.210^{-7}\ p_T^3$$

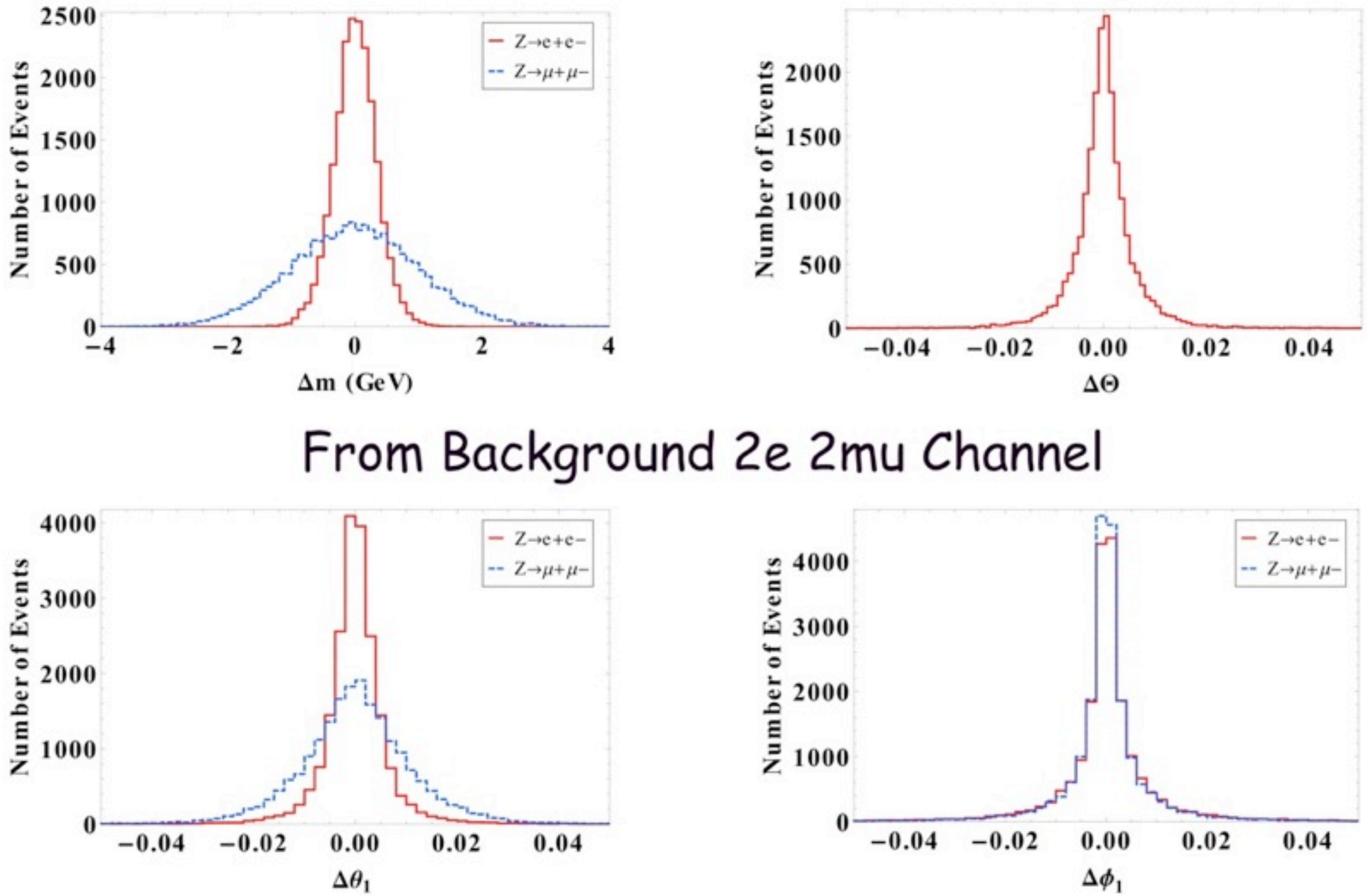
following the CMS TDR

Smearing

- The constant term in the electron energy resolution may be optimistic, so we tried doubling it; this did not significantly affect our results.

- We do not include the different reconstruction efficiencies for electrons and muons, though these would certainly need to be included in a more sophisticated analysis.

Effect of Smearing



Acceptance, Cuts

- We impose the cuts

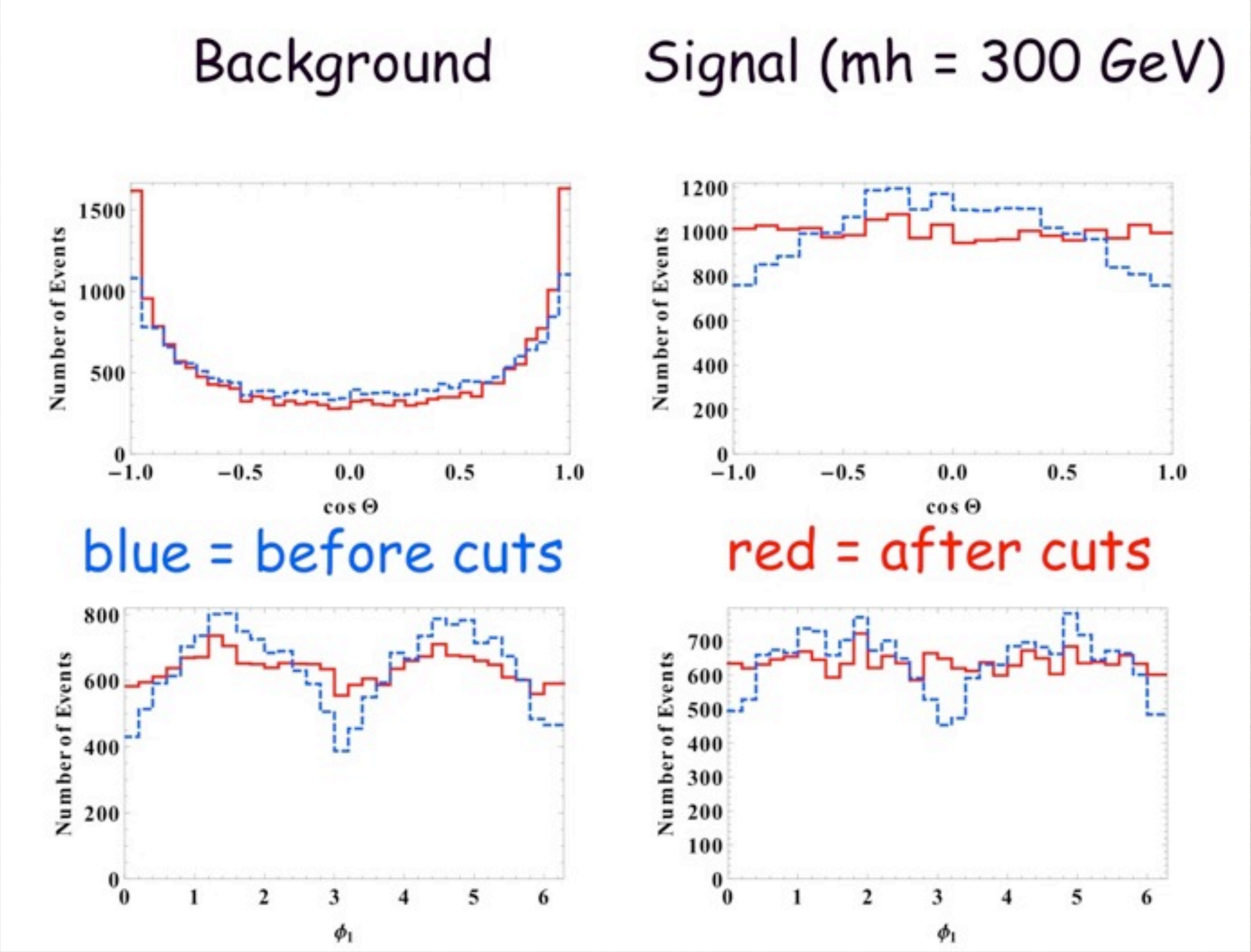
$$|p_T| \geq 10 \quad \text{GeV}$$

$$|\eta| \leq 2.5$$

and consider only background events in the range

$$150 \quad \text{GeV} \;\leq\; \hat{s} \;\leq\; 450 \quad \text{GeV}$$

Effect of Smearing, Acceptance and Cuts



Background and signal generically more similar than before cuts, etc.

Significance- Procedure

- To understand how much use of the MEM would increase significance we performed 10,000 pseudo-experiments for (each of) m_h = 175, 200, 220, 250, 300, and 350 GeV.

- Each pseudo-experiment had a number of signal and background events chosen from a Poisson distribution.

- Each pseudo-experiment involves events from 3 channels, 4e, 2e2μ, and 4μ.

Number of Events in Pseudo-experiment

	m_h(GeV)	σ(fb)	ϵ	$\langle N \rangle$
Signal	175	0.218	0.512	0.279
	200	1.26	0.594	1.87
	220	1.16	0.625	1.81
	250	0.958	0.654	1.57
	300	0.714	0.701	1.25
	350	0.600	0.708	1.06
Background	-	8.78	0.519	11.4

For 2.5 fb^{-1}, 2e 2μ channel

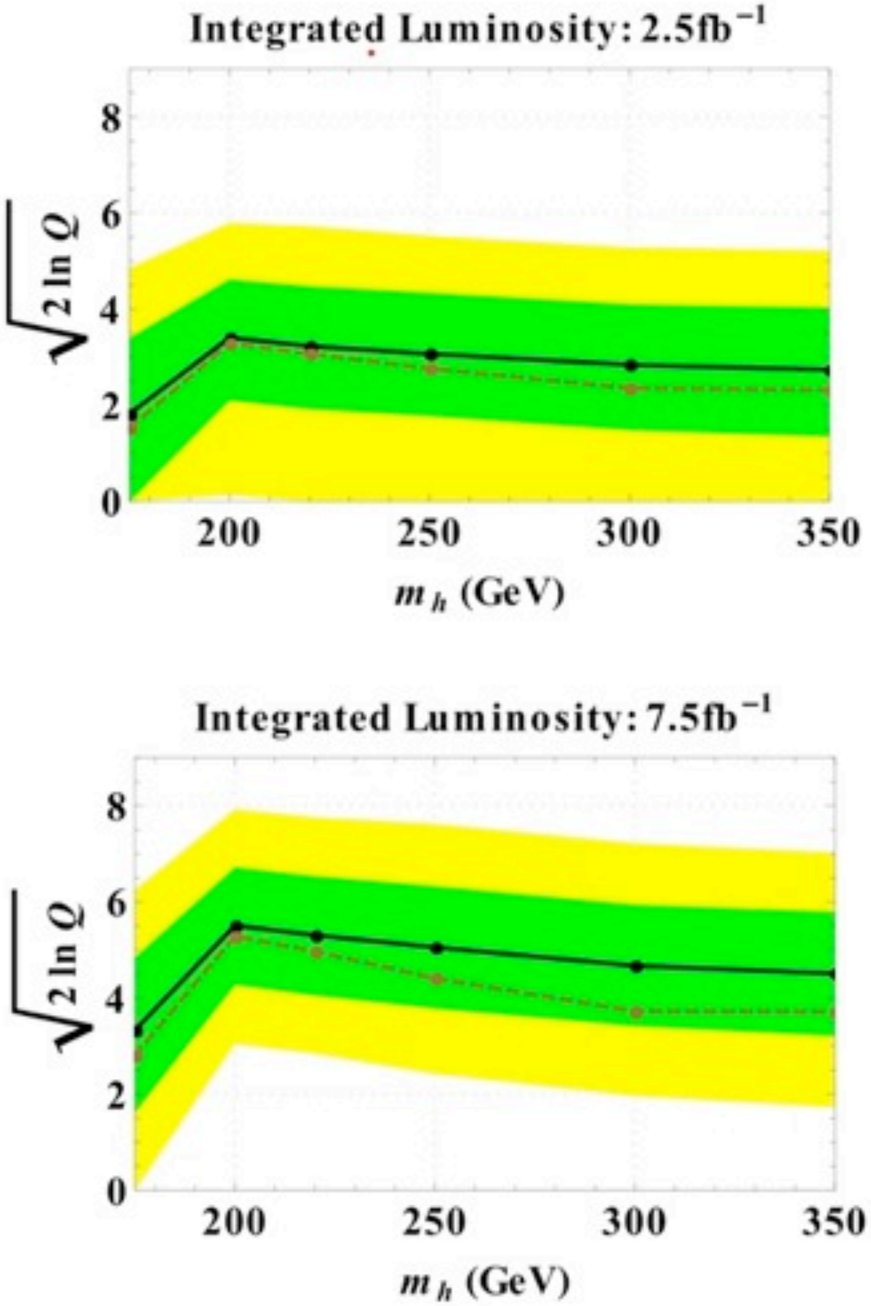
Significance- Procedure

- For each pseudo-experiment we maximized the likelihood with respect to yield, f, and the Higgs mass, using the same Higgs mass for all 3 channels.

- Obtained a measure of significance from the ratio between the likelihood of the signal + background hypothesis to the likelihood for the background only hypothesis.

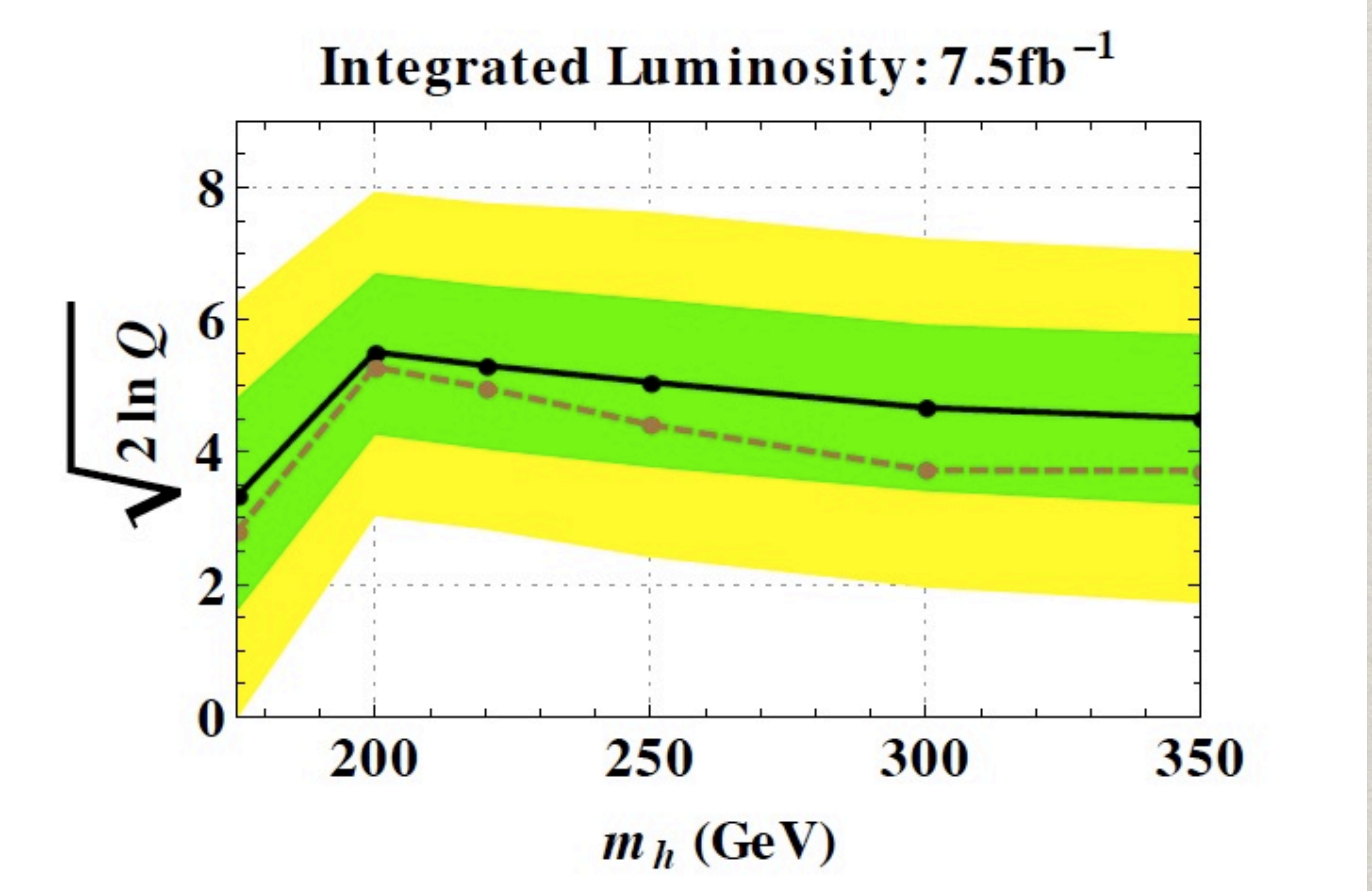
- Compare significance obtained with MEM to that obtained with a likelihood which is only a function of mass information (M_1, M_2, m_{4l})

Significance



MEM central value
1 sigma band
2 sigma band
Invariant masses only

Significance



Significance

- Difference in significance greatest at higher Higgs masses, when background is dominated by
  $(\pm,\mp)$ ZZ production helicity amplitudes

- Signal is (always) all
  $(\pm, \pm)$ and $(0,0)$
  ZZ production helicity amplitudes

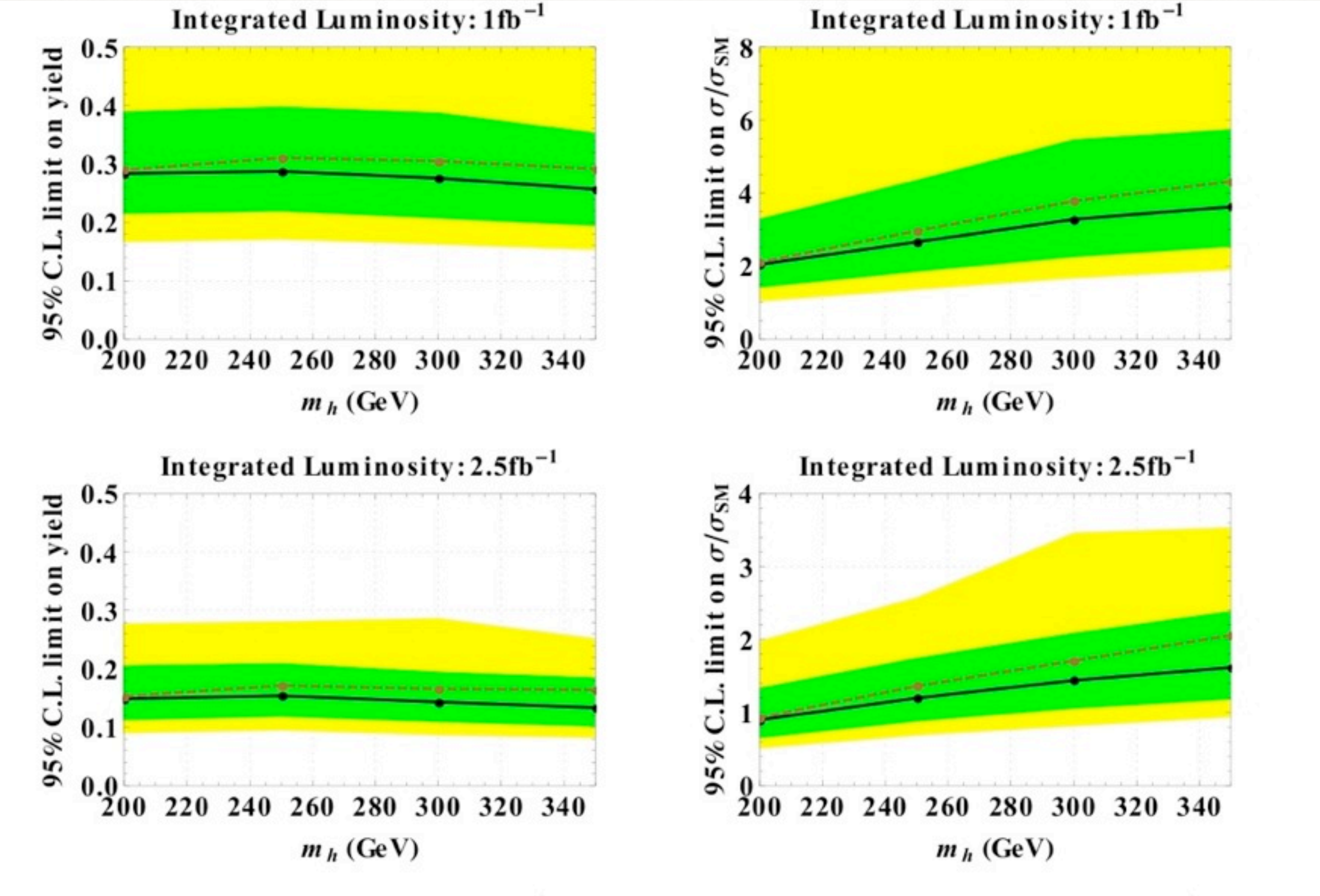
Limits- Procedure

- As noted above, we treat the expected number of signal and background events as a parameter to be obtained from data
- To really set limits, one should use some information about overall signal, background cross sections
- Interesting, and simpler in our setup, to see how well one can do with only the yield parameter, f. The corresponding limit should be weaker, as overall cross section information is not used.

Limits- Procedure

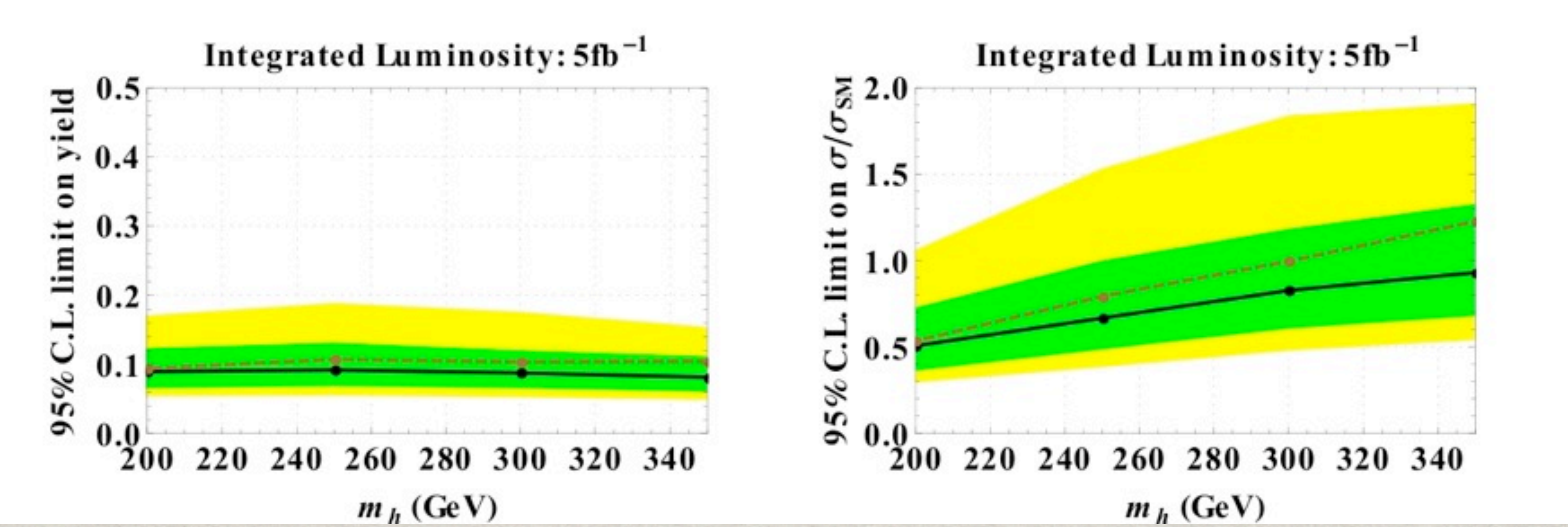
- Performed 1000 pseudo-experiments for Higgs masses of 200, 250, 300, and 350 GeV.
- Obtained 95% confidence limits on yield parameter f.
- Demand f be the same for each channel (4e, 2e2μ, 4μ)
- Can translate these into limits on signal cross section assuming background perfectly known (for illustrative purposes only).

Limits



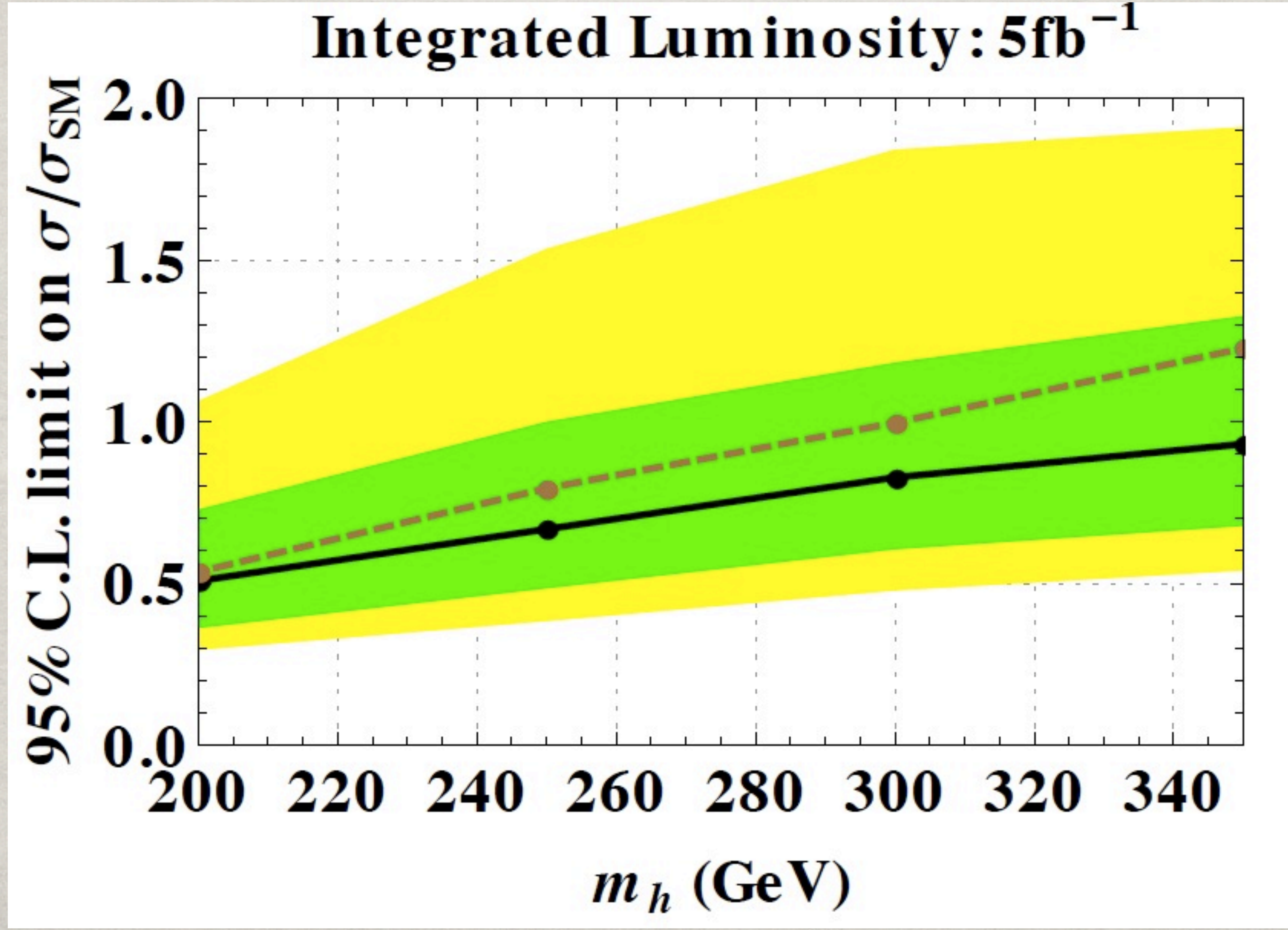
Integrated Luminosity: 1fb^{-1}
95% C.L. limit on yield
0.0
0.1
0.2
0.3
0.4
0.5
200 220 240 260 280 300 320 340
m_h (GeV)
Integrated Luminosity: 1fb^{-1}
95% C.L. limit on $\sigma/\sigma_{\mathrm{SM}}$
0
2
4
6
8
200 220 240 260 280 300 320 340
m_h (GeV)
Integrated Luminosity: 2.5fb^{-1}
95% C.L. limit on yield
0.0
0.1
0.2
0.3
0.4
0.5
200 220 240 260 280 300 320 340
m_h (GeV)
Integrated Luminosity: 2.5fb^{-1}
95% C.L. limit on $\sigma/\sigma_{\mathrm{SM}}$
0
1
2
3
4
200 220 240 260 280 300 320 340
m_h (GeV)

Limits



Integrated Luminosity: 5fb⁻¹
95% C.L. limit on yield
0.5
0.4
0.3
0.2
0.1
0.0
200 220 240 260 280 300 320 340
m_h (GeV)
Integrated Luminosity: 5fb⁻¹
95% C.L. limit on σ/σ_SM
2.0
1.5
1.0
0.5
0.0
200 220 240 260 280 300 320 340
m_h (GeV)

Limits



Integrated Luminosity: 5fb^{-1}
95% C.L. limit on σ/σ_{SM}
2.0
1.5
1.0
0.5
0.0
200
220
240
260
280
300
320
340
m_h (GeV)

Further (Future?) Complications

- We used a LO cross sections and considered only the zero-jet bin
  - though we did use K-factors for signal and background production
- Interesting to see there are e.g. particular masses where NLO changes things.
- In particular gg -> ZZ backgrounds could interfere with signal production,
  changing the angular correlations,
  though this effect is probably small.

Further (Future?) Complications

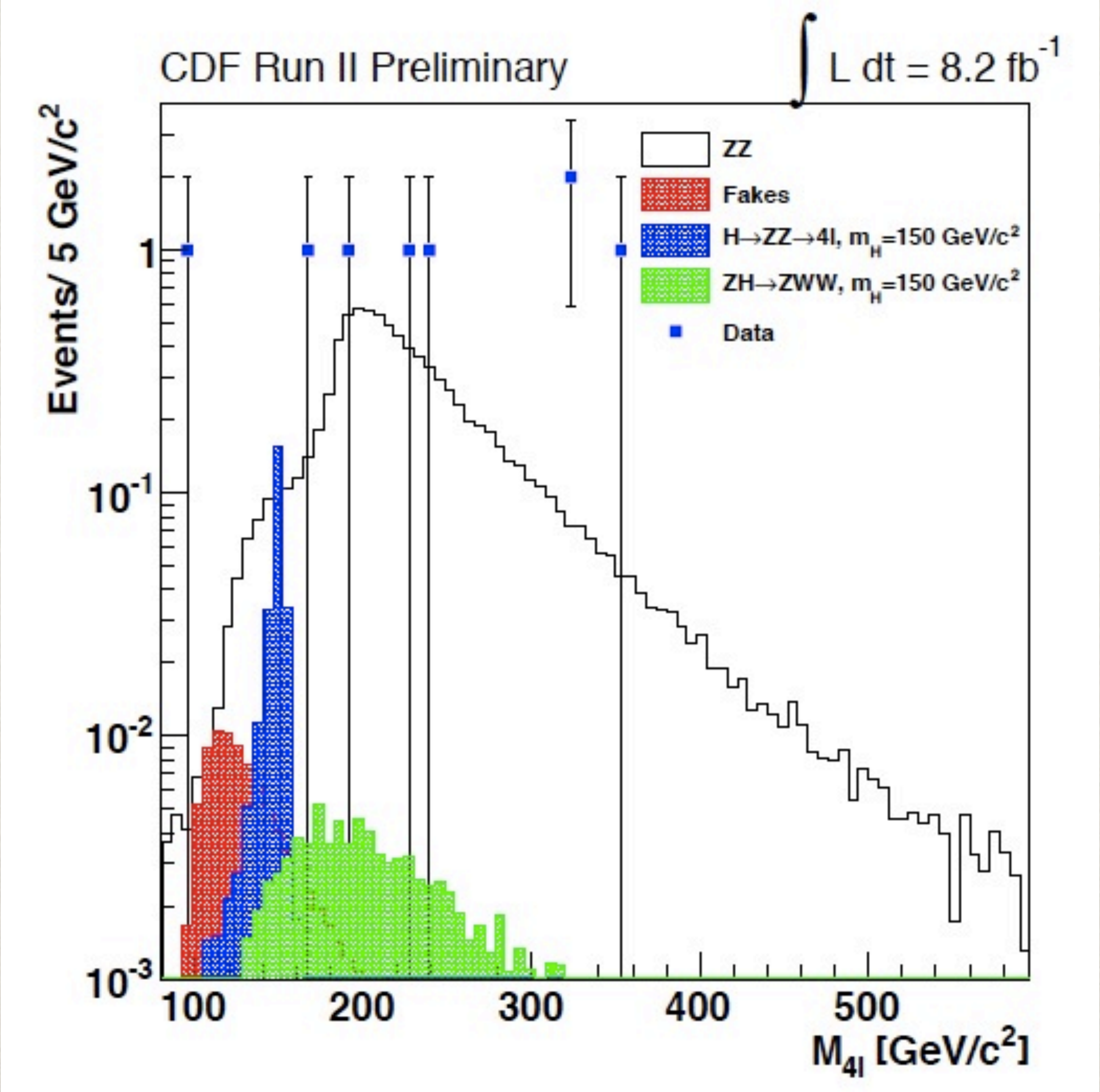
- Also it would be interesting to consider more jets.

- Alwall and Freitas studied a method for approximating multi-jet matrix elements when using the MEM
  (Phys.Rev. D83 (2011) 074010).
  It would be interesting to quantify how well this approach works for this process.

- Sometimes additional jets could give useful information

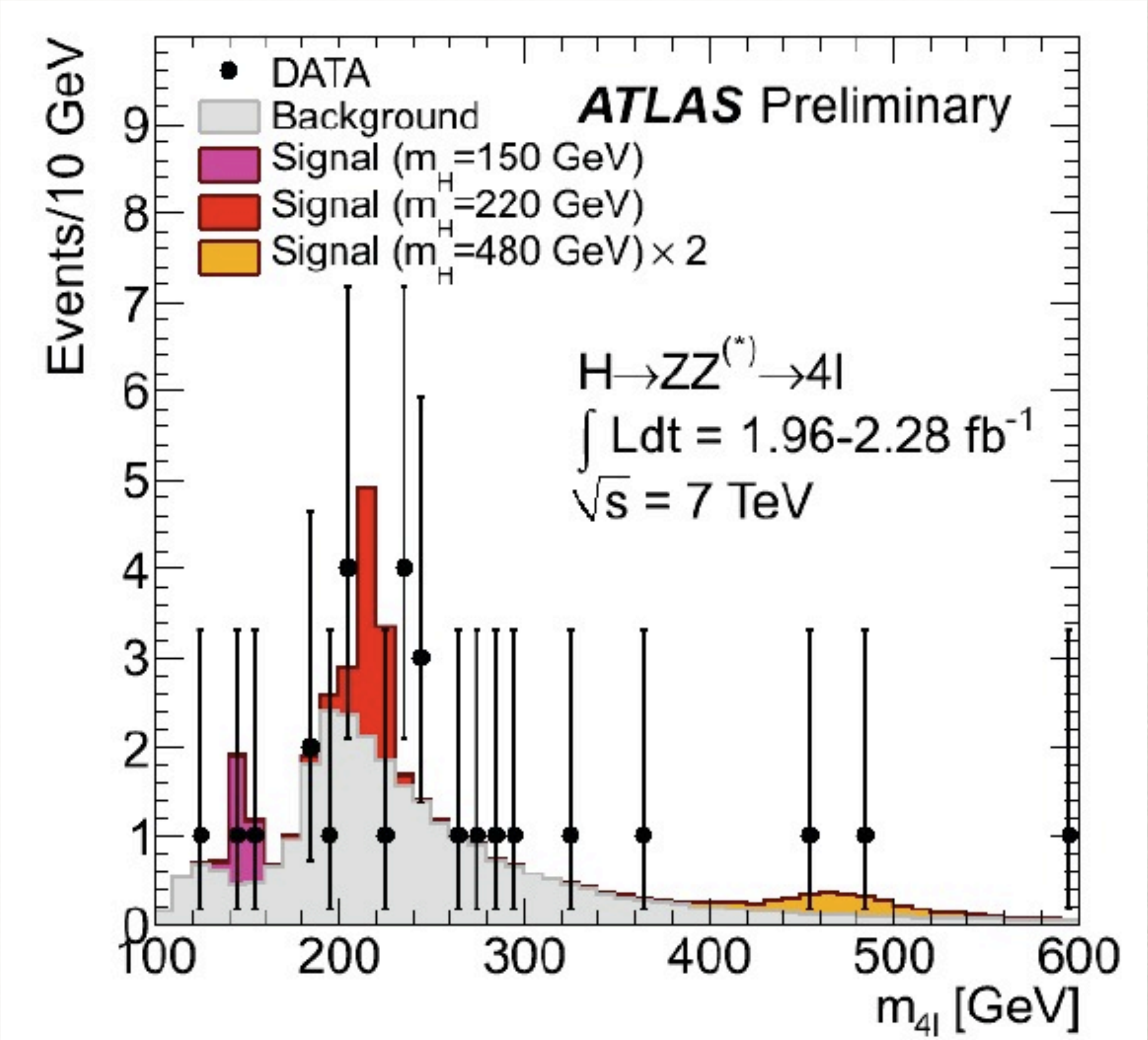

AMBULANCE
AMBULANCE

CDF ZZ -> 4l Excess

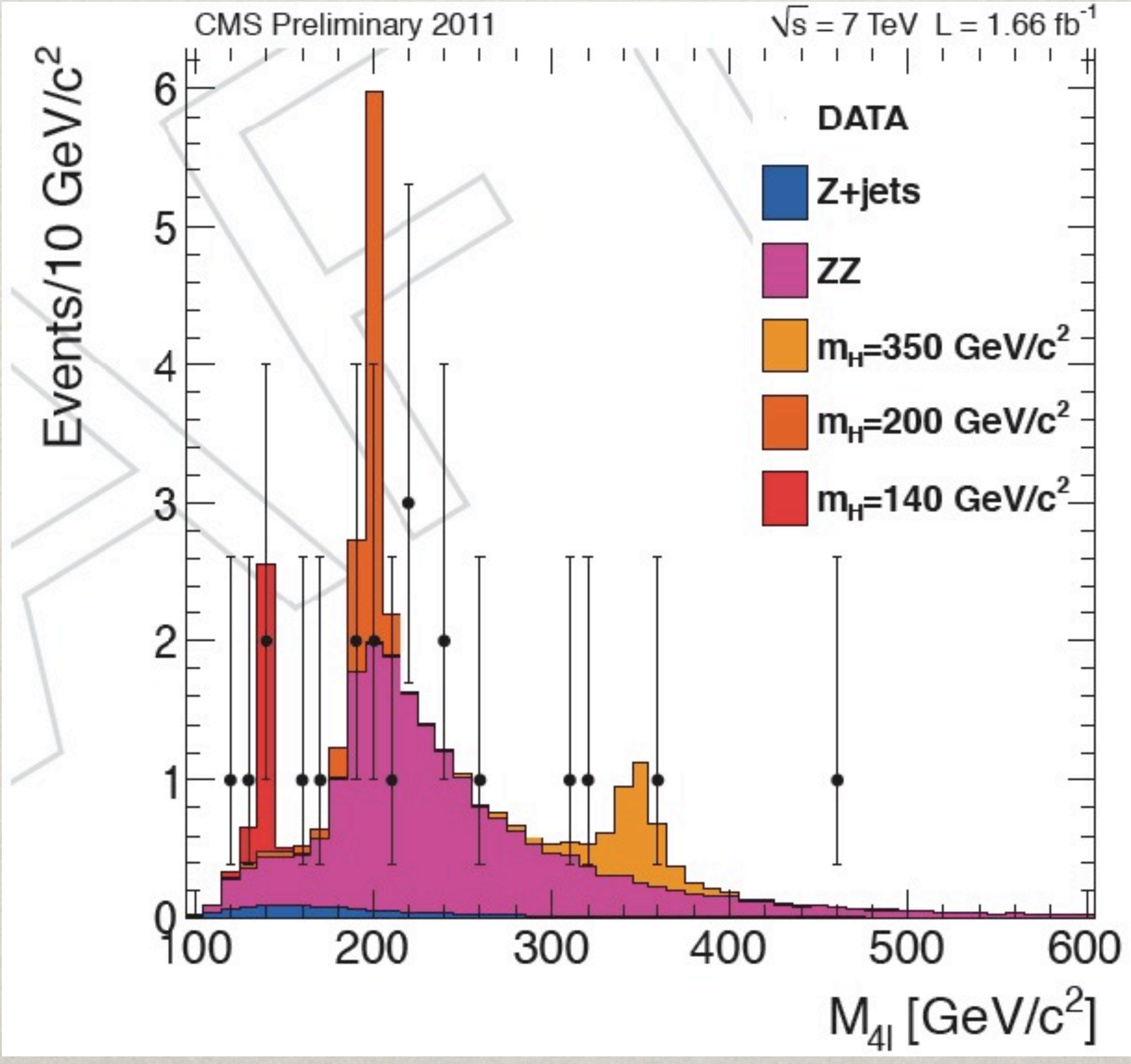


2 events with $M_{4l} \approx 300$ GeV

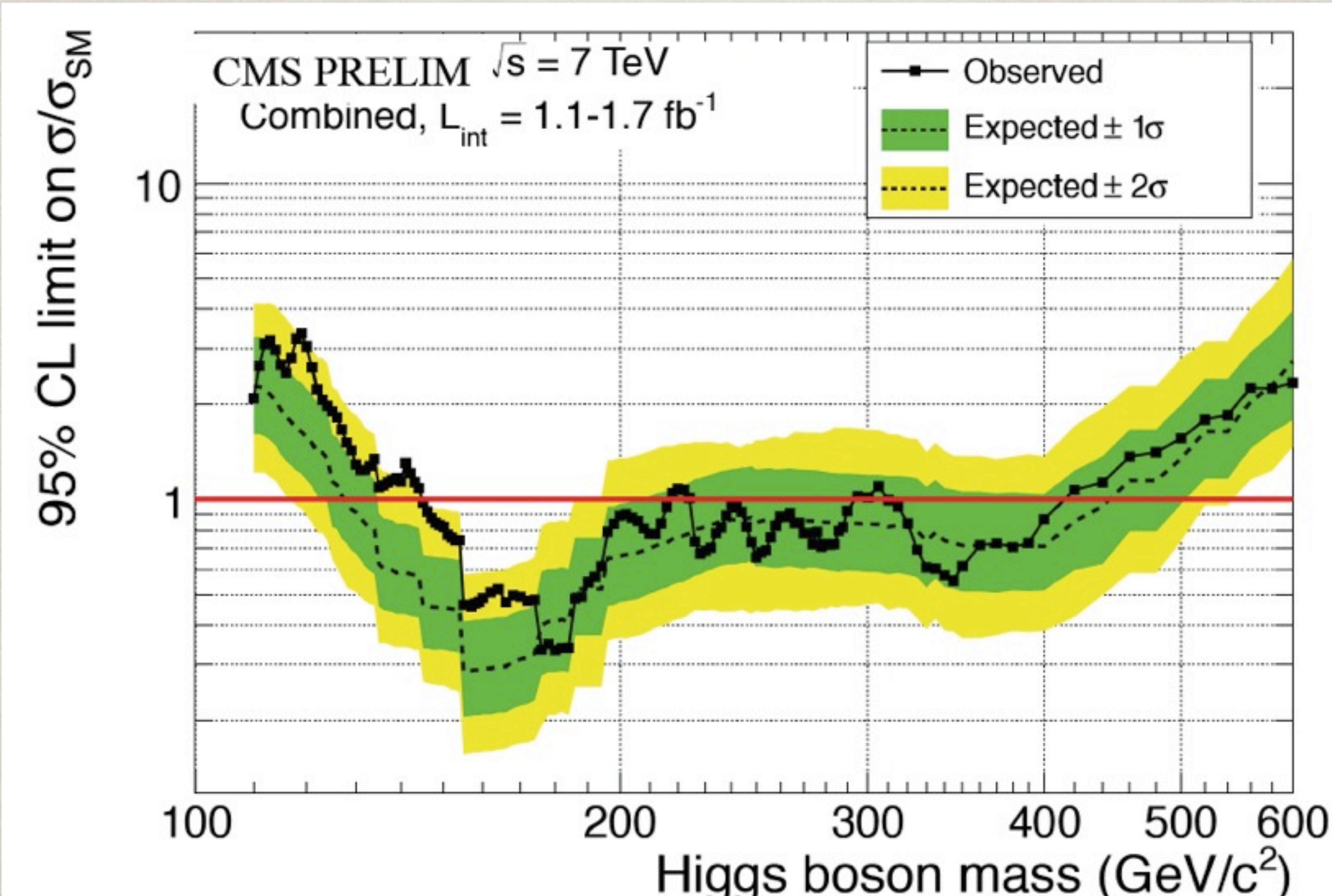
ATLAS ZZ -> 4L


DATA
Background
Signal (m_H=150 GeV)
Signal (m_H=220 GeV)
Signal (m_H=480 GeV) × 2
ATLAS Preliminary
$H \rightarrow ZZ^{(*)} \rightarrow 4l$
$\int Ldt$ = 1.96-2.28 fb^{-1}
$\sqrt{s}$ = 7 TeV
Events/10 GeV
m_{4l} [GeV]
0
1
2
3
4
5
6
7
8
9
100
200
300
400
500
600

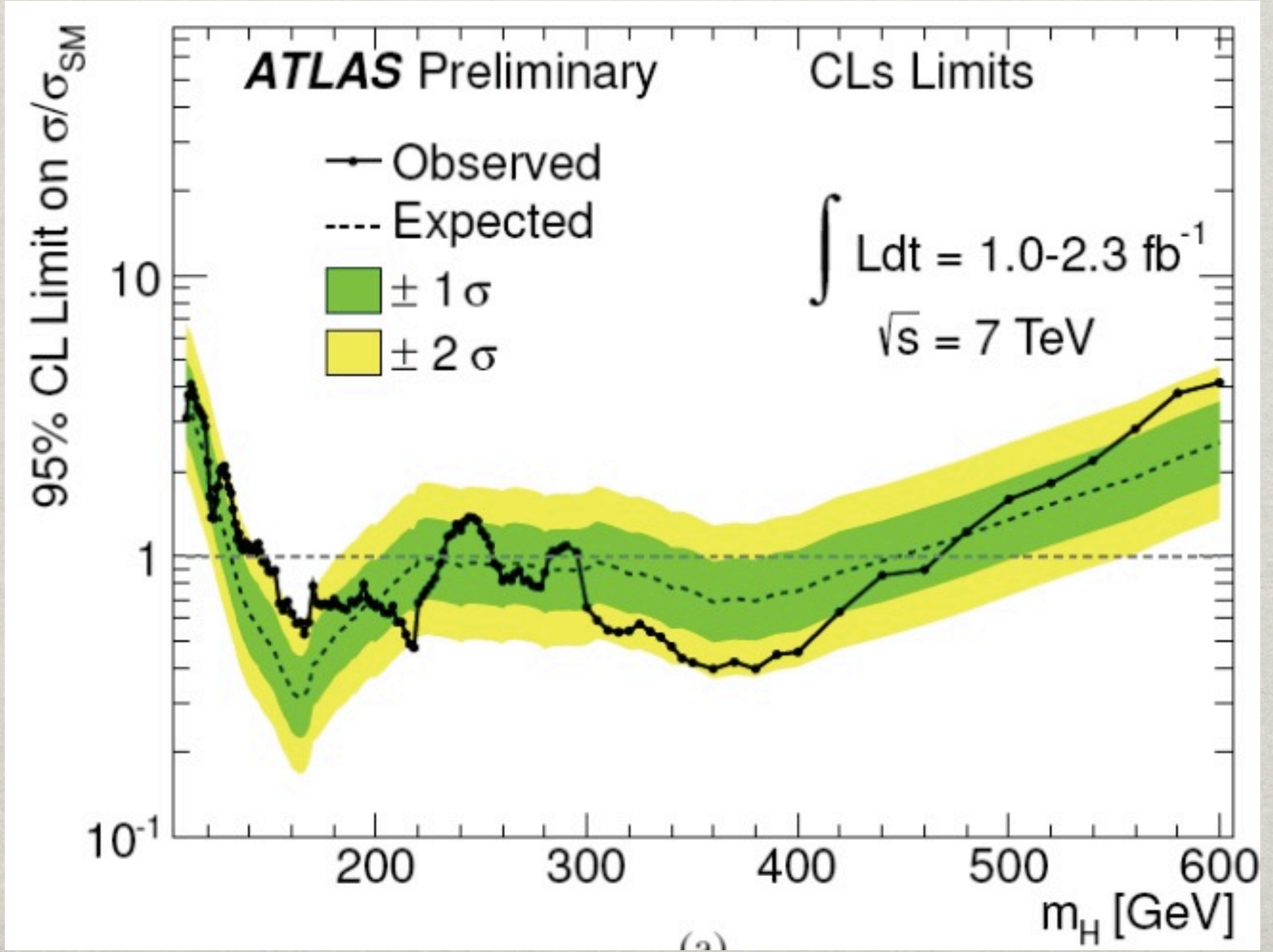
CMS ZZ -> 4L


CMS Preliminary 2011
$\sqrt{s}$ = 7 TeV L = 1.66 fb^{-1}
Events/10 GeV/c^2
DATA
Z+jets
ZZ
m_H=350 GeV/c^2
m_H=200 GeV/c^2
m_H=140 GeV/c^2
0
1
2
3
4
5
6
100
200
300
400
500
600
M_{4l} [GeV/c^2]

CMS Higgs Exclusion



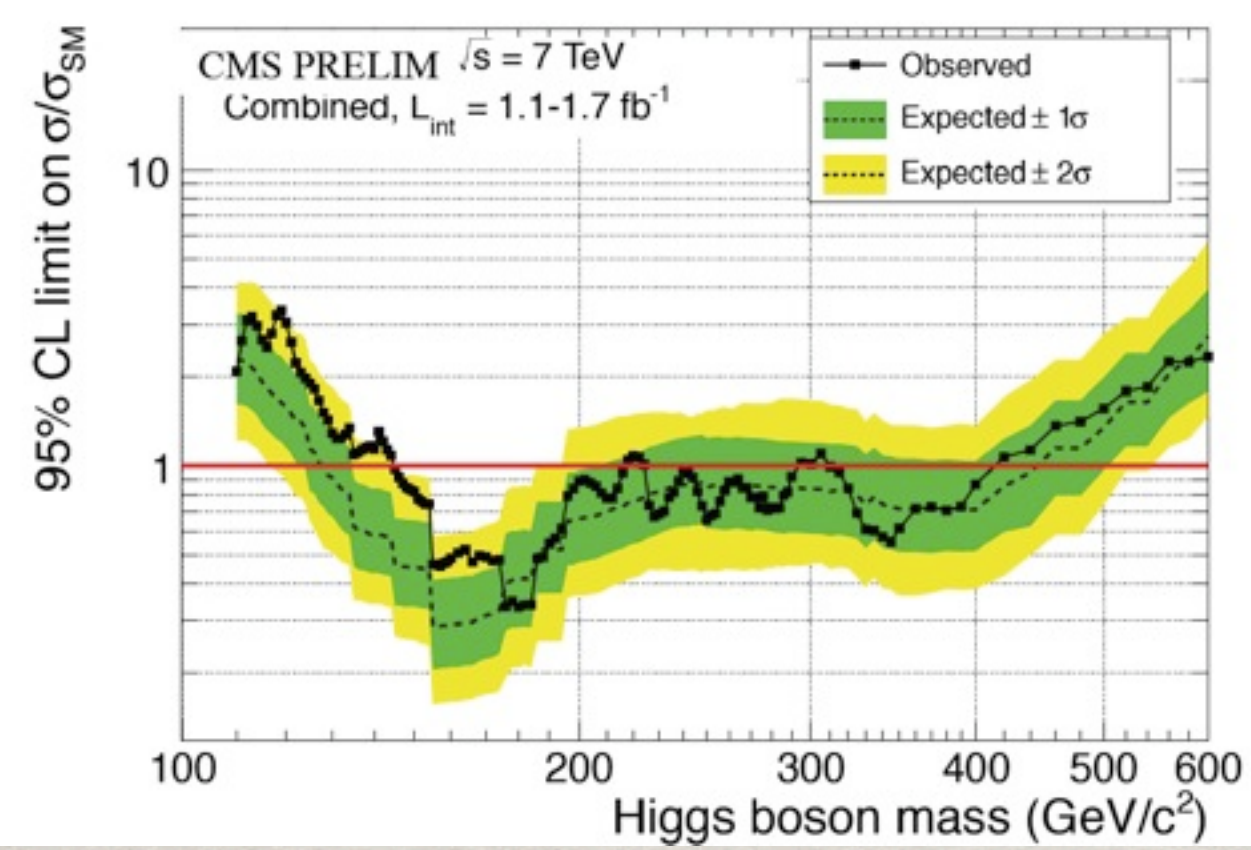
ATLAS Higgs Exclusion

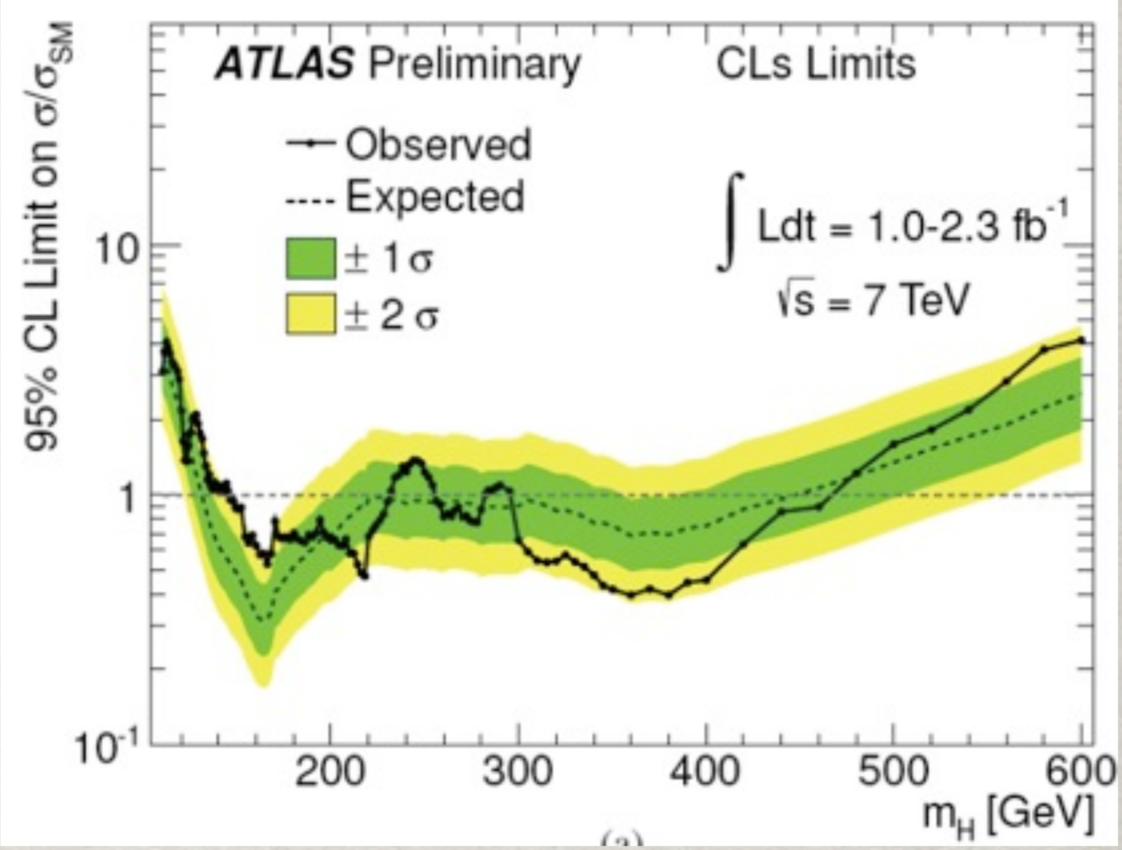


Experimental Situation

- Heavy SM Higgs nearly ruled out in combination
  - (though not quite in a few places)
- However, we would like to be able to set stricter limits in every channel, to exclude (or discover) non-SM Higgses.
- So still important to study how to increase sensitivity for heavy Higgs.
- Working with experimentalists from CMS and ATLAS to implement analyses like ours.

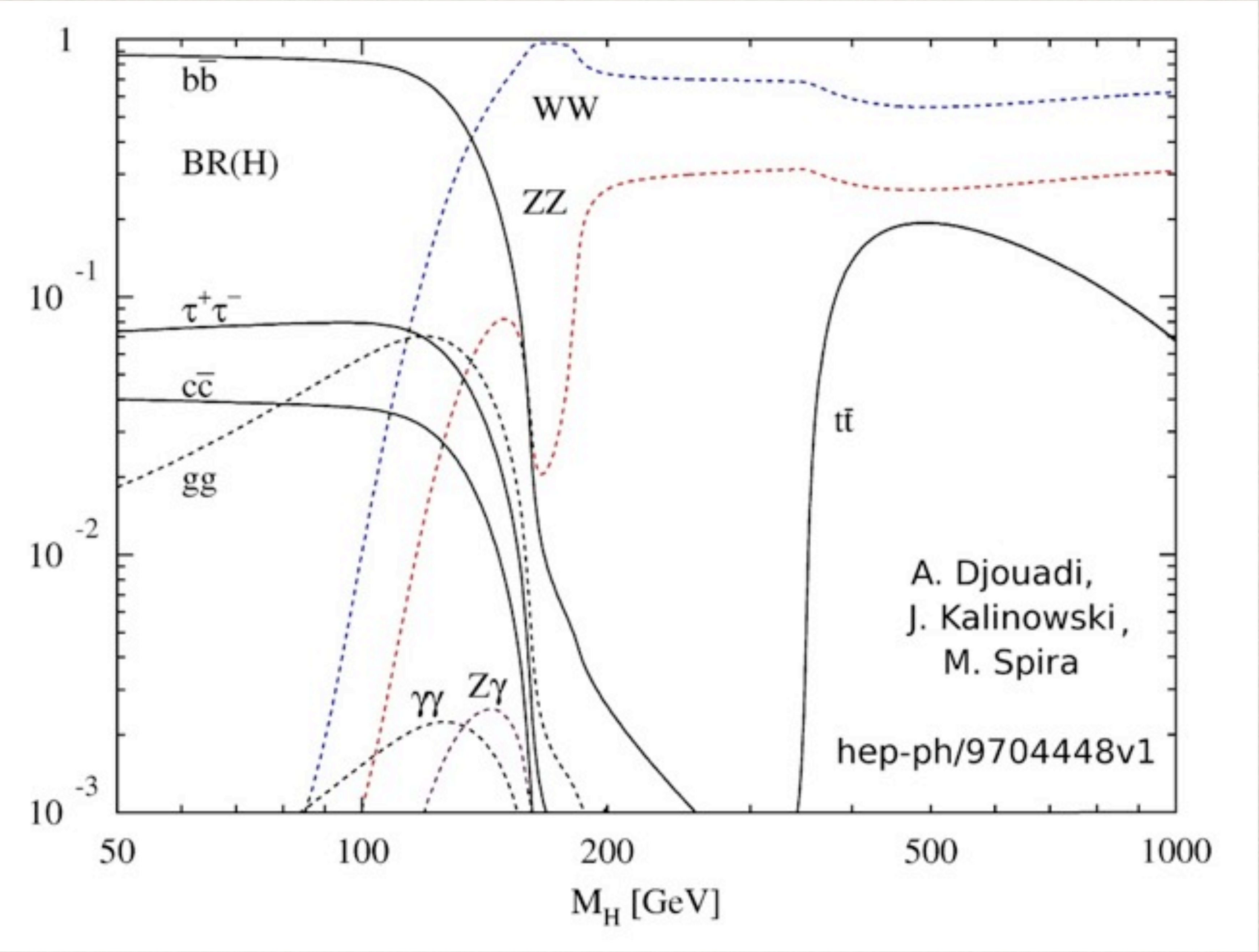
Future Directions





- Observed limits being higher than expected limits for a broad range of lower Higgs masses suggests studying ~130 - 140 GeV Higgs

Future Directions



- ZZ* is not too bad for somewhat lower mass (increased Higgs production somewhat compensates for lower BR)

Future Directions

- BR(h -> Zγ) always less than BR(h -> ZZ*)
- However, demanding leptonic decays for each channel enhances
  Zγ relative to ZZ* (-> 4l) by ~16.
- So BR(h -> Zγ -> llγ) > BR(h -> ZZ* -> 4l) for mh < ~125 GeV. It is within a factor of 2 for mh < ~145 GeV.

Future Directions

- This channel should provide important additional information for spin discrimination in this mass ranges.
- Might also help for discovery, though the number of events is small.
- Important to quantify the extent to which this is the case.
- Working with Wai-Yee Keung, Ian Low, and Pedro Schwaller to do this.

Conclusions

- A study of using the MEM for Higgs discovery in the "golden mode" of h -> ZZ -> 4l suggests that ~10 - 20% increases in sensitivity may be obtained.
- Experimental interest now.
- Continued usefulness in future to set limits on additional? non-SM Higgses with smaller cross section times branching ratio.
- Work in progress on how useful the MEM would be in ZZ^* at lower Higgs masses, also in $Z\gamma$.
- **Exciting times for Higgs Physics!**